

Design of beams due to Torsional Moment.

Twisting Moment



تصميم الكمرات على عزم الإلتواء

نسألكم الدعاء

If you download the Free **APP. RC Structures**  on your smart phone or tablet, you will be able to play illustrative movies For any paragraph that has a QR code icon 

إذا حملت تطبيق **RC Structures**  على تليفونك المحمول أو اللوح السطحي ستستطيع أن تشغل أفلام شرح للمقاطع التي تحتوى على رمز 

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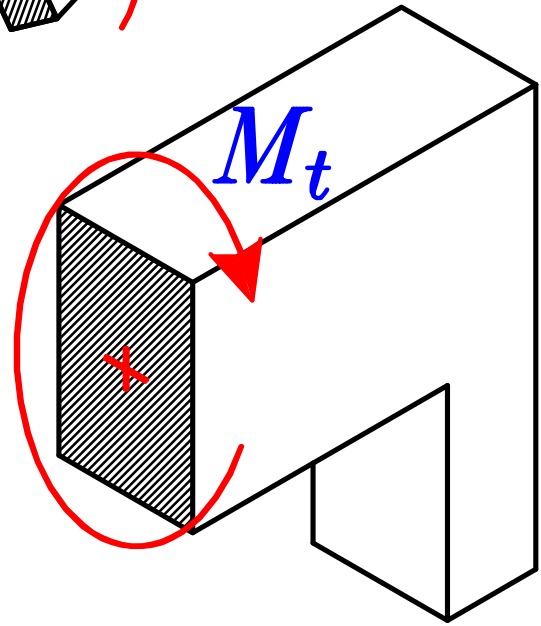
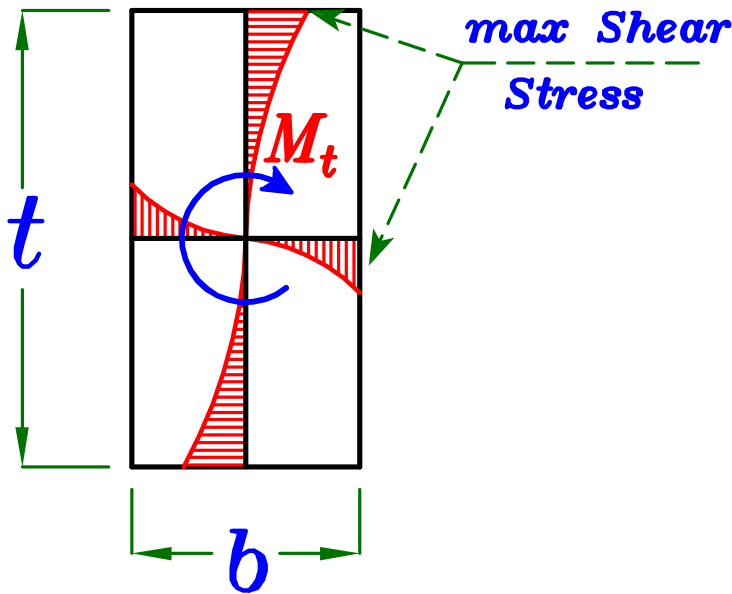
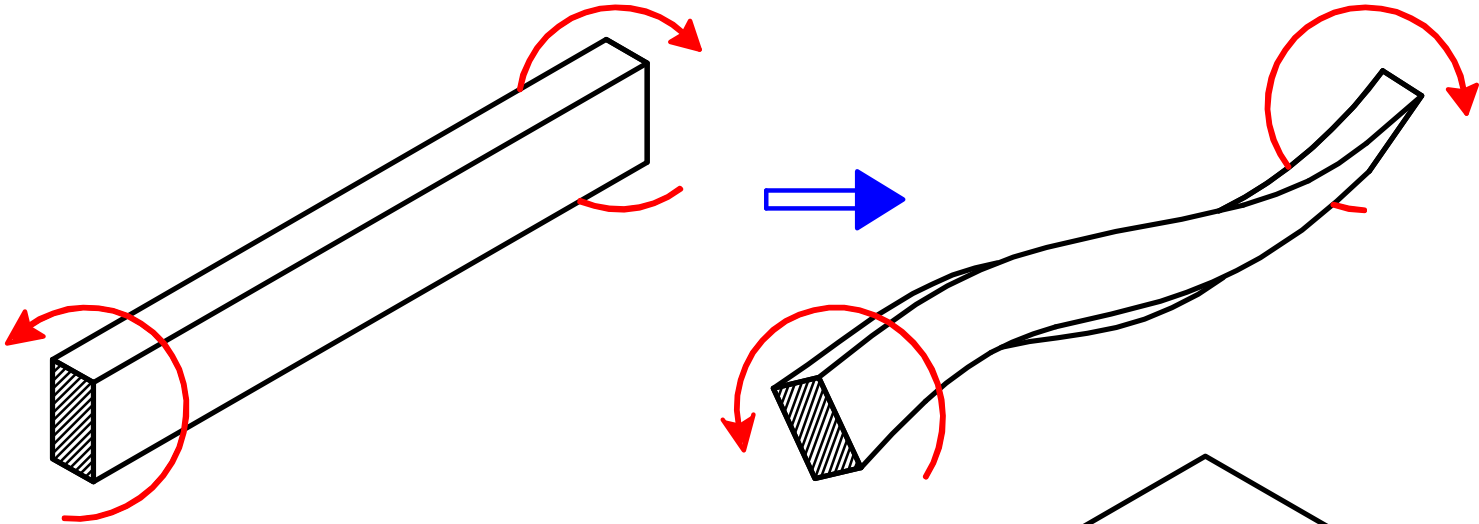
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Torsion Moment.



عزم الالتواء

عزم الالتواء هو عزم ينتج عندما تدور الكمره حول محورها .



تنتج إجهادات القص *Shear Stresses* على الكمرات عن طريق

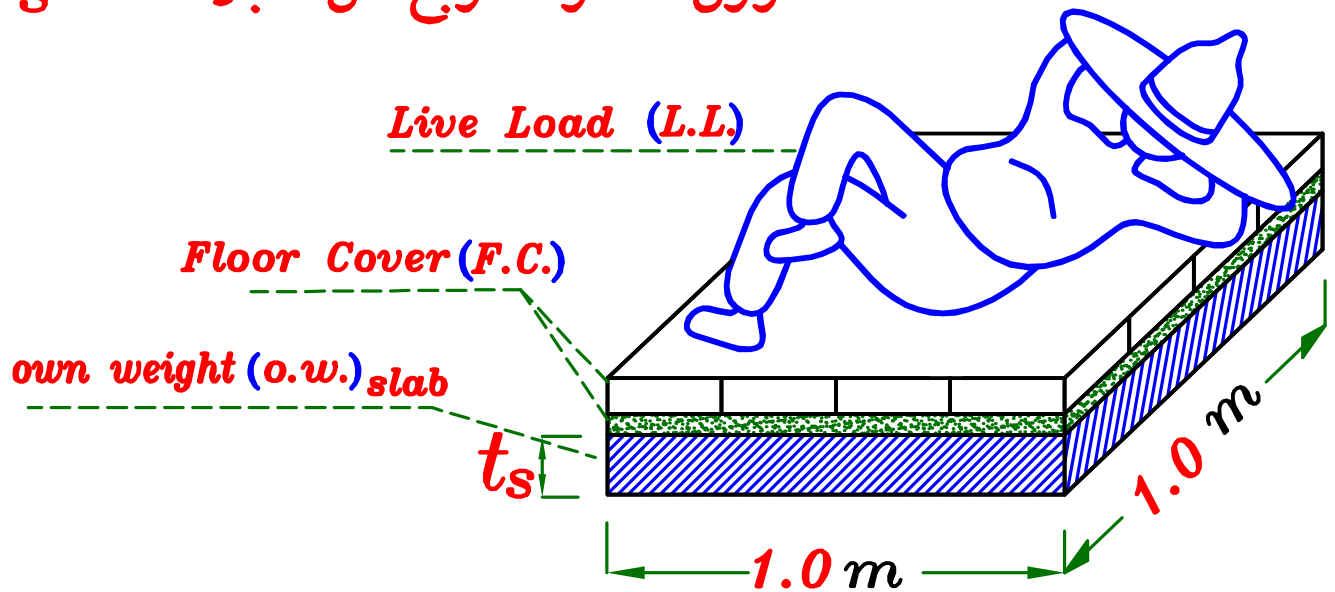
- ① *Shear Force (Q)* ----- قوى القص
- ② *Torsional Moment (M_t)* ----- عزم الالتواء

لذا سيتم تصميم الكانات لتحمل كلاً من *Shear Force & Torsional Moment*

Strip in the Slab.

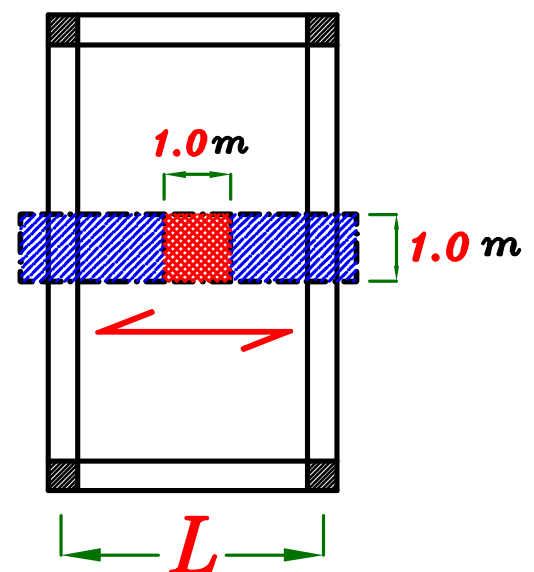
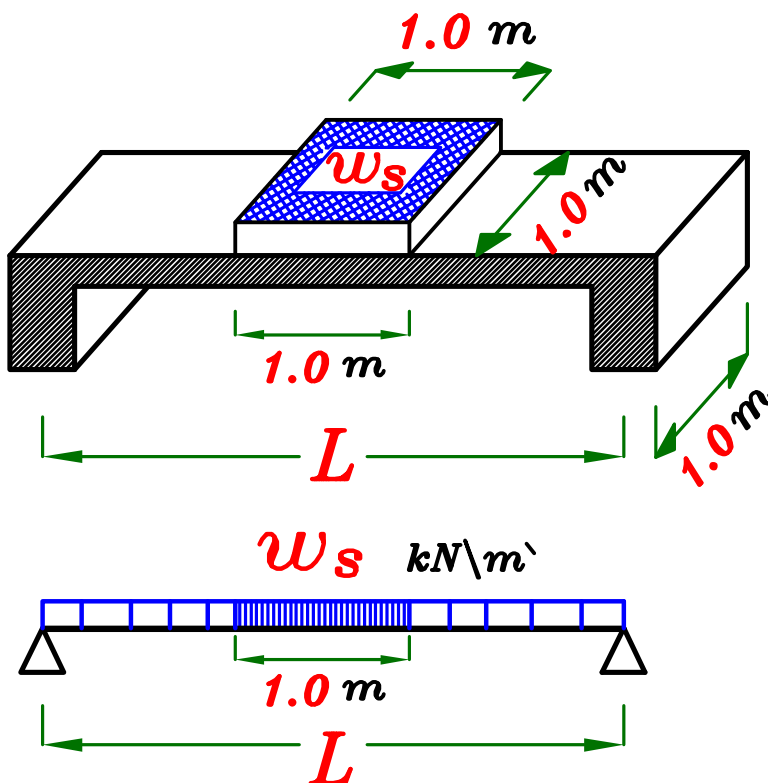
شريحة في البلاطة .

w_s = وزن المتر المربع من البلاطة



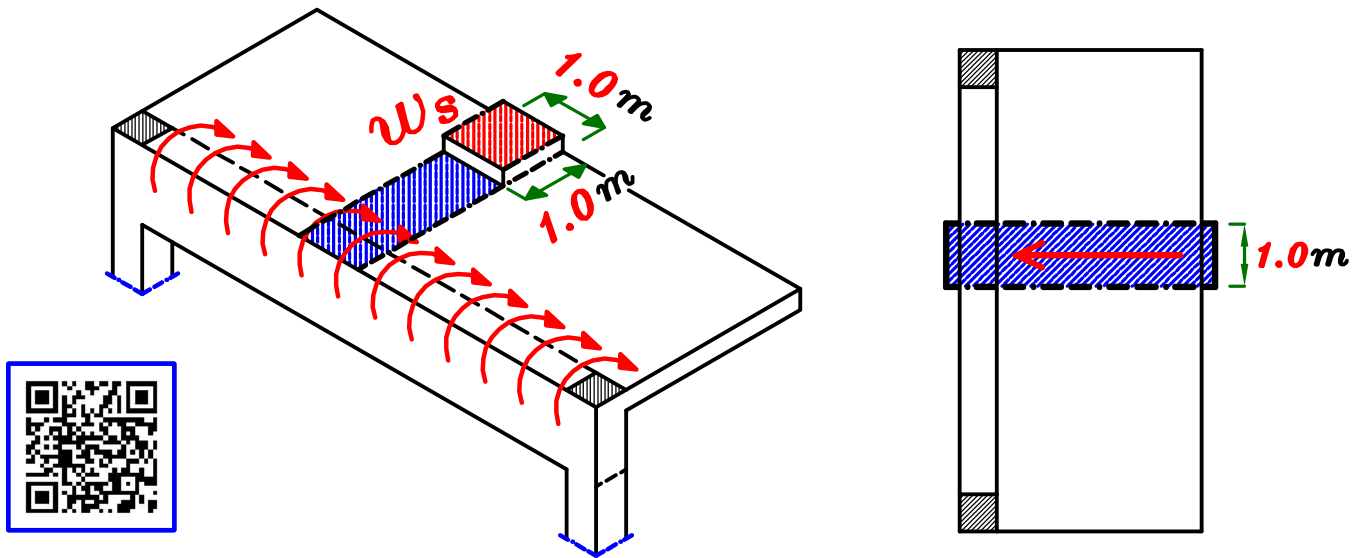
$$(w_s)_{U.L.} = 1.4 (t_s \gamma_c + F.C.) + 1.6 (L.L.) \quad kN/m^2$$

عند أخذ شريحة في البلاطة في اتجاه الـ **Load** عرضها 1،
نضع عليها حمل منتظم **uniform Load** قيمته w_s



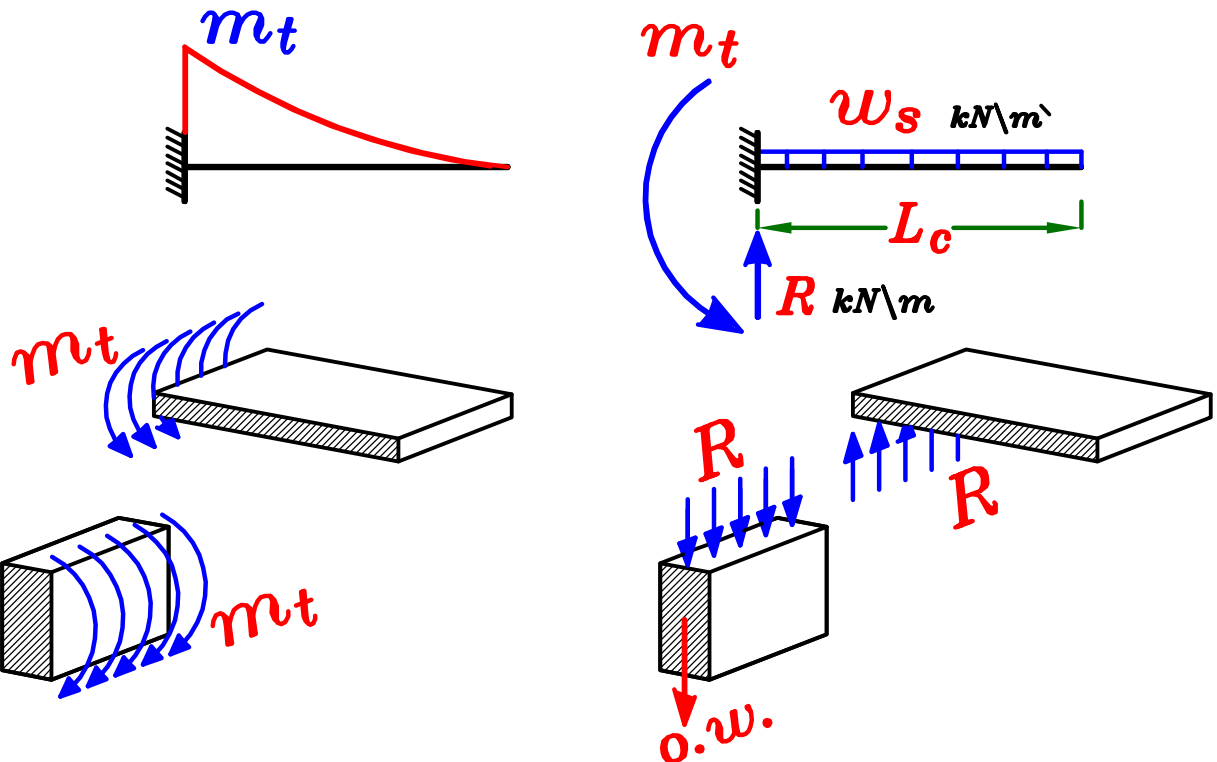
When Beam affected to Torsional Moment ?

- ١- تكون الكمره عليها **Torsion** منتظم عندما تكون شريحه البلاطه محموله على كمره واحده .
 أما اذا كانت الشريحه محموله على أكثر من كمره فلن يوجد أى **Torsion** على الكمرات .

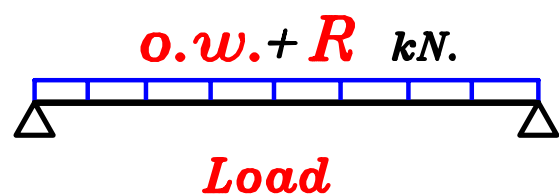
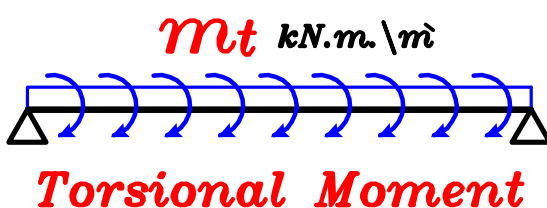


نأخذ شريحه فى البلاطه فى اتجاه ال **Load** عرضها ١, - م

Take a strip in the slab at the load direction with width 1.0m



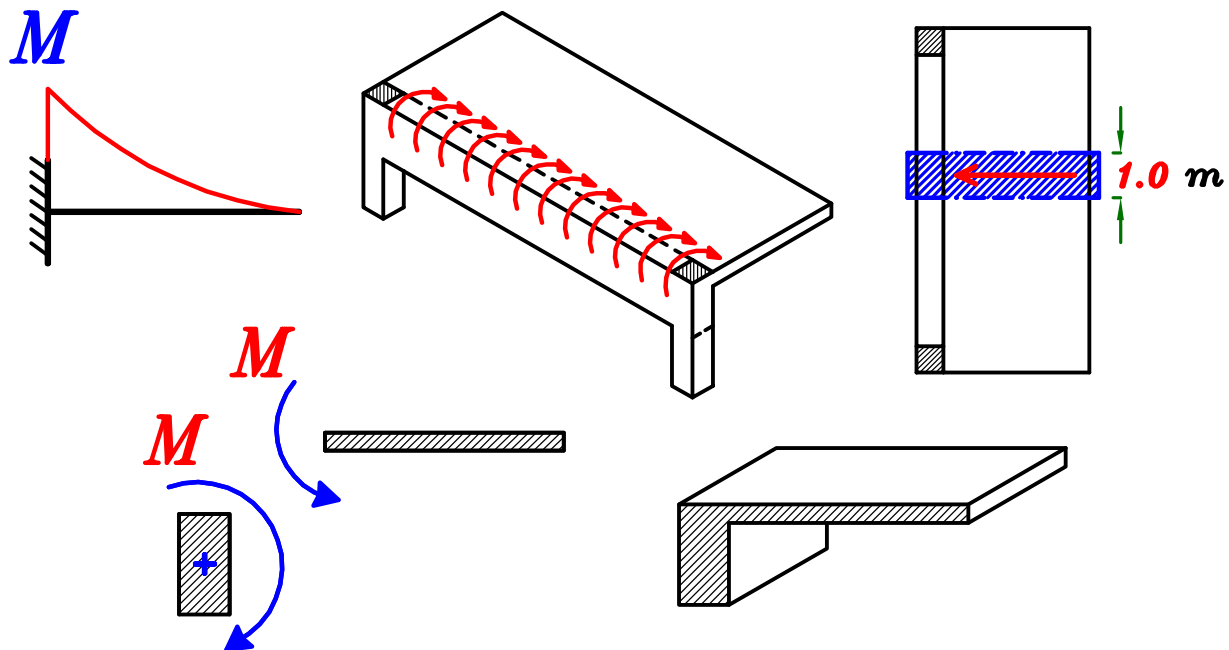
Beam.



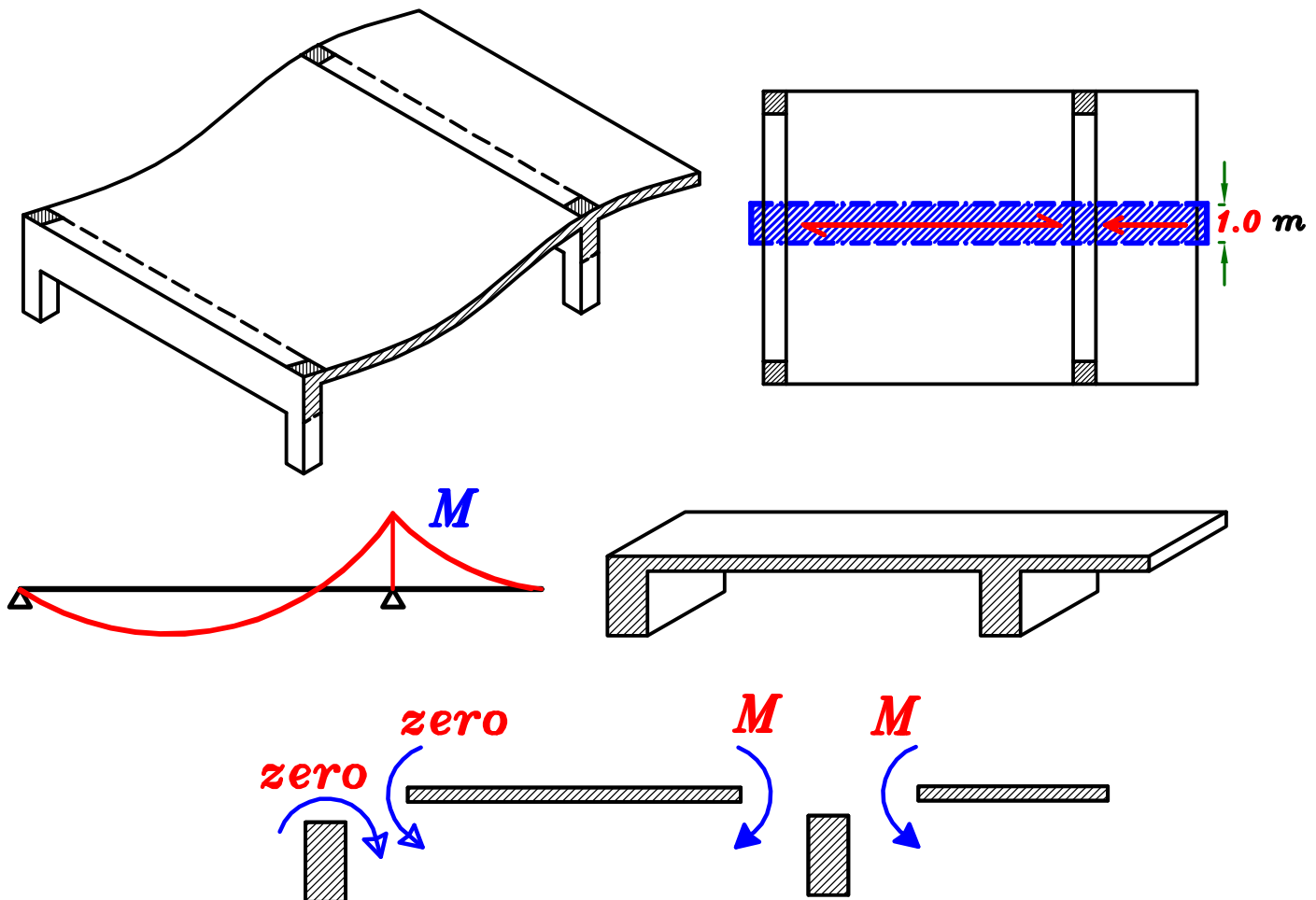
ملحوظه .

ينتقل عزم الالتواء من البلاطة الى الكمره

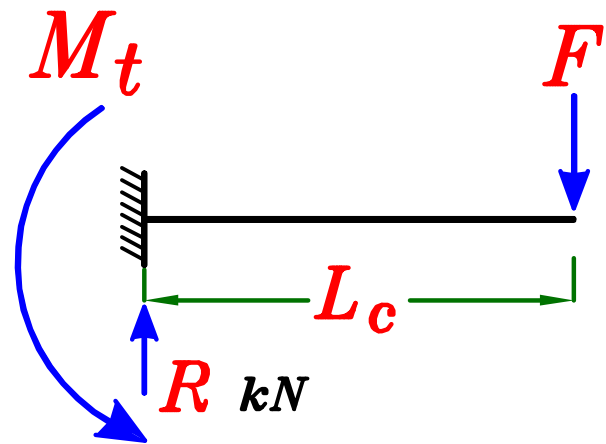
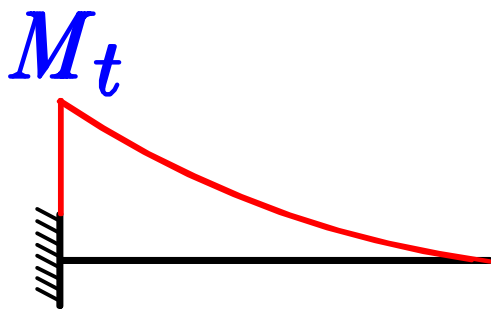
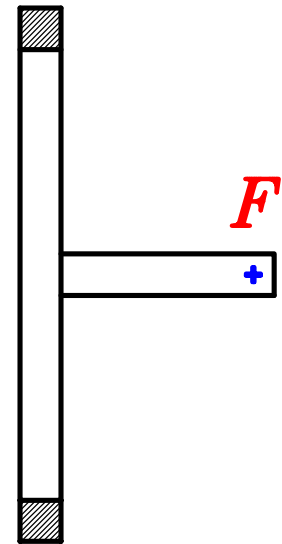
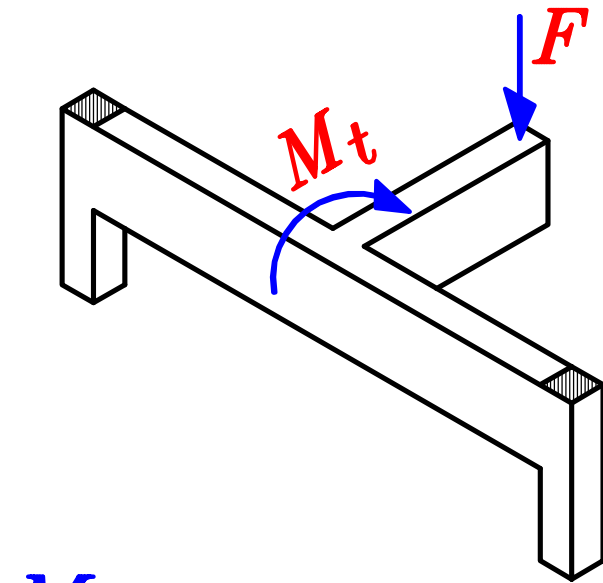
اذا كانت شريحه البلاطة محموله على كمره واحده فقط فيعمل **Torsion** على الكمره .



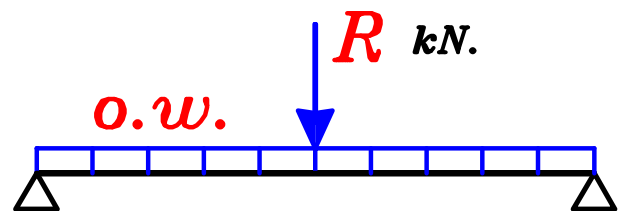
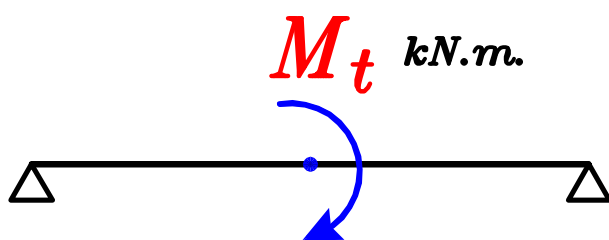
اذا كانت شريحه البلاطة محموله على اكثر من كمره لن يكون هناك **Torsion** لان العزم سينتقل من البلاطة للبلاطة المجاوره مباشره .



٢- تكون الكمره عليها **Torsion** مركز عندما تكون كمره محموله على كمره واحده فقط .



Beam.

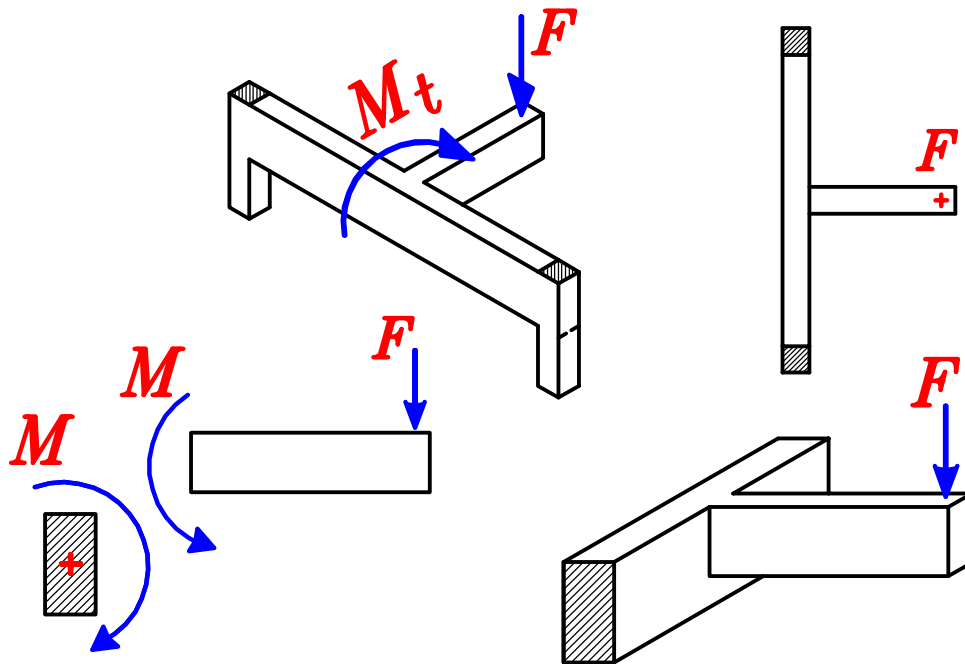


Torsional Moment

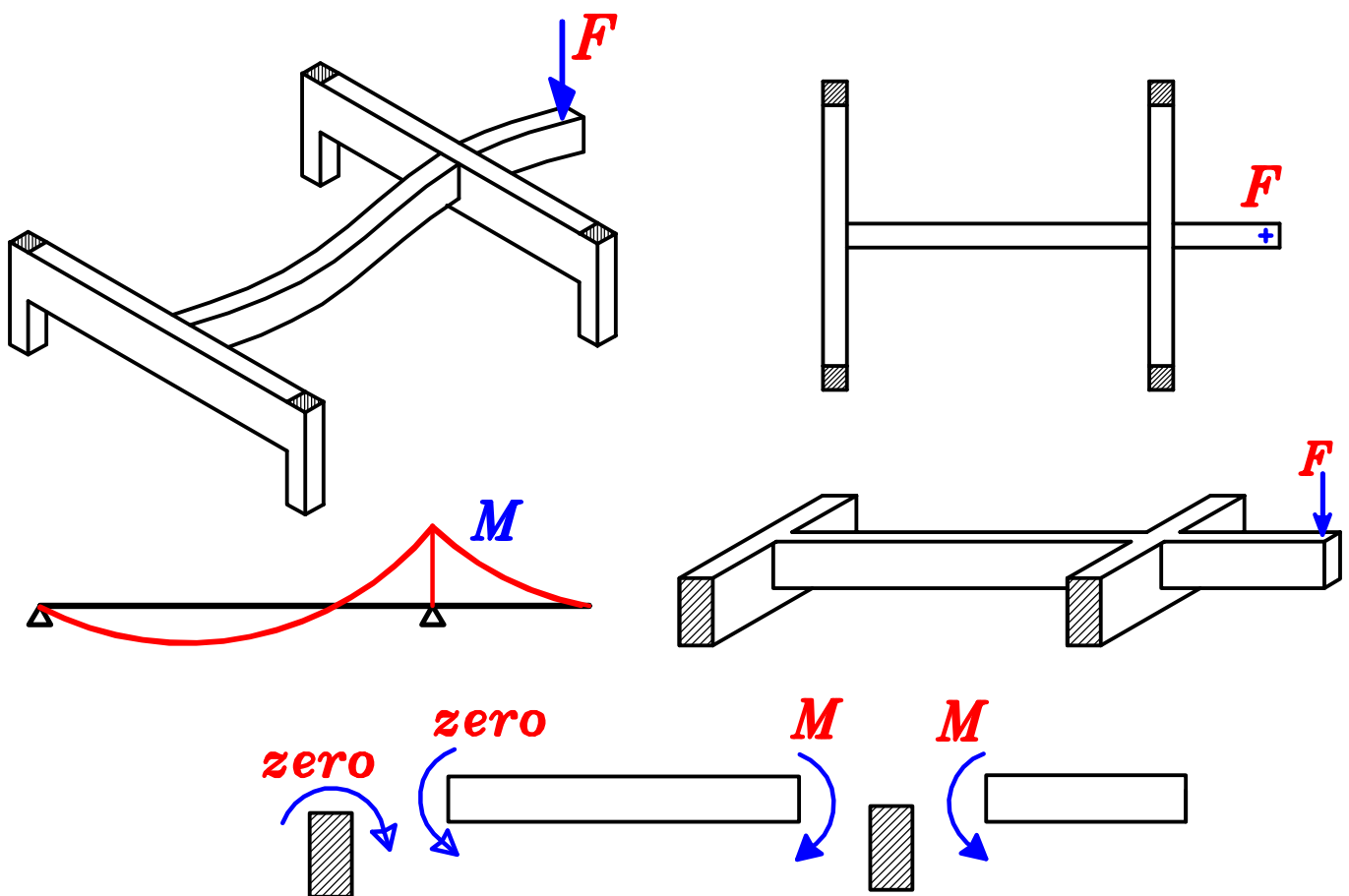
Load

ملحوظه .

ينتقل عزم الالتواء من الكمره الـ **Cantilever** الى الكمره التي تحملها .
لان الكمره الـ **Cantilever** محموله على كمره واحده فقط .

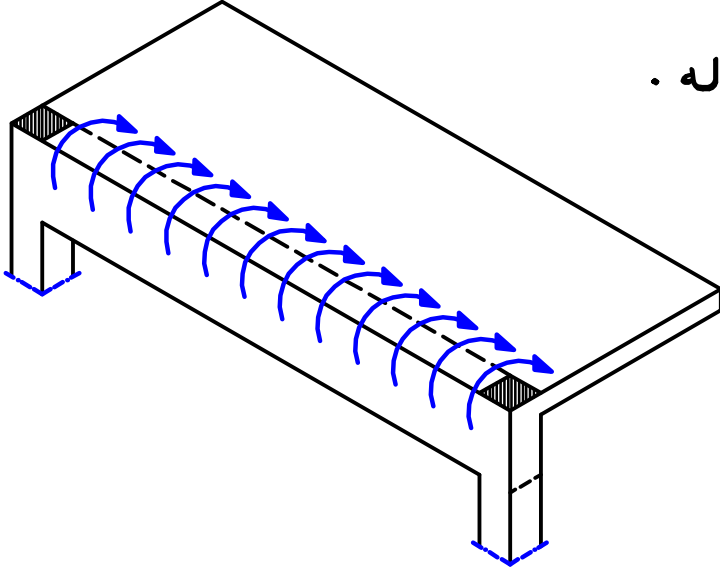


اذا كانت الكمره محموله على اكثر من كمره سينتقل العزم من الـ **cantilever** الى الباقيه التاليه دون ان ينقل **torsion** على الكمره الحامله .



Equilibrium Torsion (Primary Torsion) .

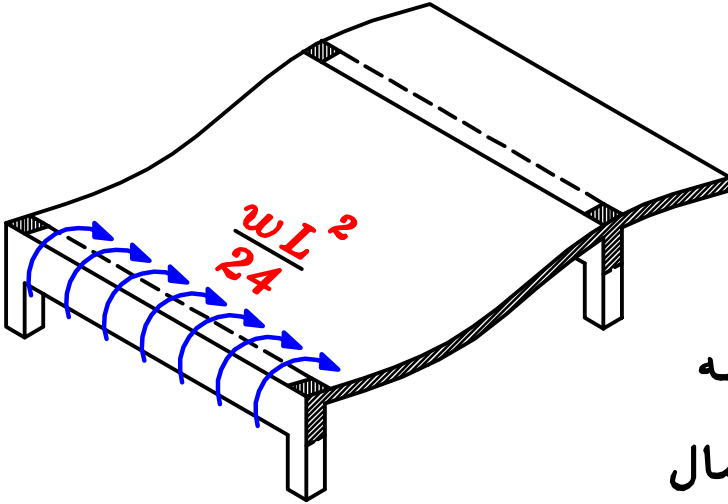
و هي الحالة التي يكون ال **Torsional Moment** أساسى لعمل اتزان للكمرة و لا يمكن اهماله .



و فى هذه الحالة لا نستطيع أن نقلل من قيمه ال **Torsional Moment** عن طريق التسليح او اعاده توزيع الاحمال و فى هذا النوع يجب ان يوضع تسليح ليقاوم ال **Torsional Moment** و لا يمكن اهماله .

Compatibility Torsion (Secondary Torsion) .

و هي الحالة التي يكون ال **Torsional Moment** غير أساسى لعمل اتزان للكمرة .



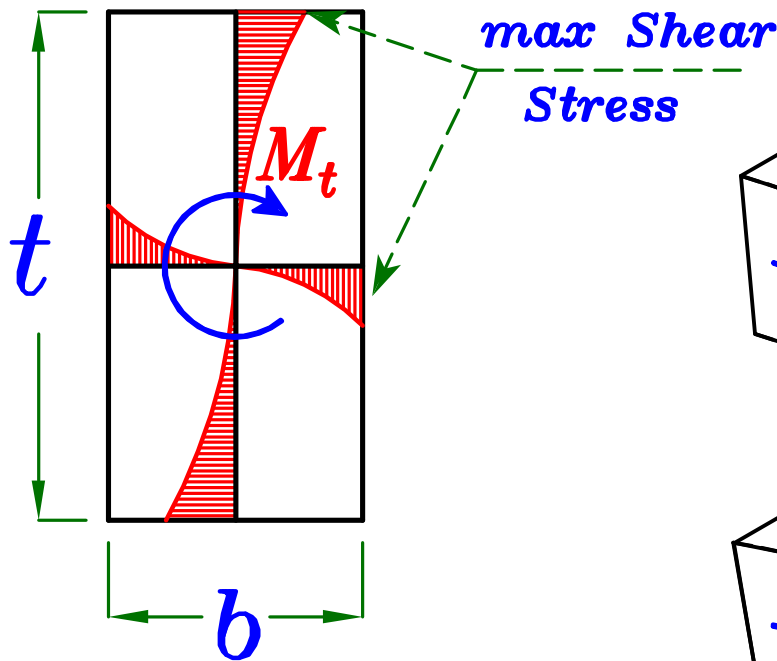
و فى هذه الحالة نستطيع أن نقلل من قيمه ال **Torsional Moment** أو نعمله عن طريق التسليح او اعاده توزيع الاحمال

و فى هذا النوع ممكن اهمال وضع تسليح ليقاوم ال **Torsional Moment** حيث عدم وضع تسليح لل **Torsional Moment** سيؤدى لشروخ فى الخرسانه و بالتالى اعاده توزيع العزوم و الغاء ال **Torsional Moment** .

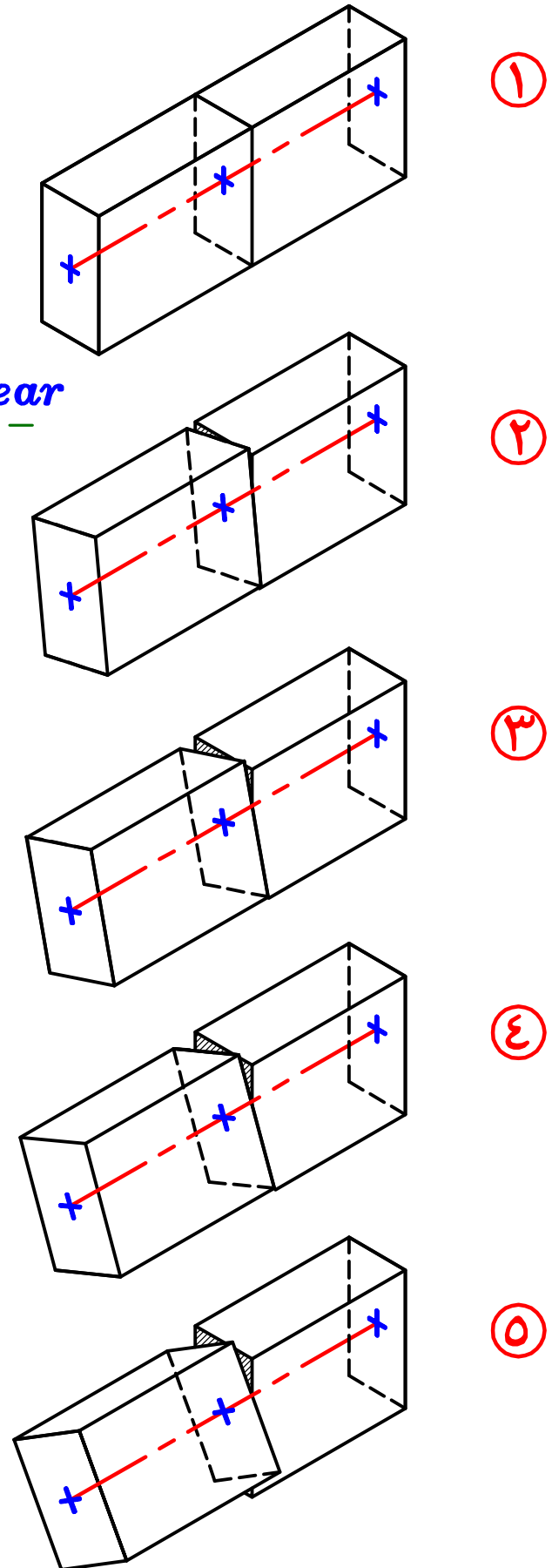
فى هذا الملف سيتم دراسته و تسليح ال **Equilibrium Torsion** فقط

Shear Stress due to Torsional Moment.

يتسبب ال **Torsional Moment** في وجود اجهادات قص ناتجة عن دوران الكمره حول محورها .
و ذلك بسبب الاحتكاك الناتج بين سطحين متلامسين احدهما يدور بزاويه أكثر من الاخر .

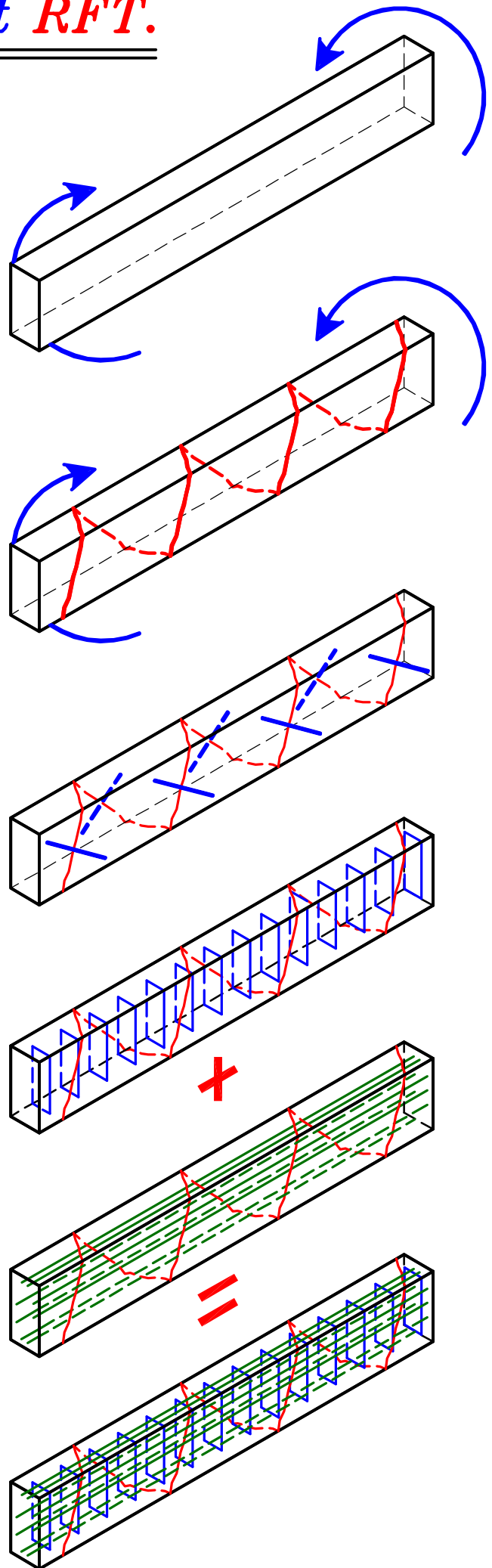


Shear Stress Distribution

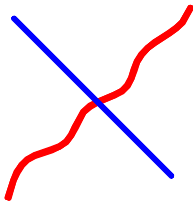


Torsional Moment RFT.

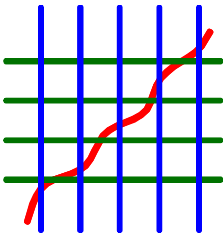
عند حدوث **Torsion** على الكمره



يكون شكل الشروخ
زاويه ميل الشرخ 45°
و يكون على المحيط الخارجى للكمره



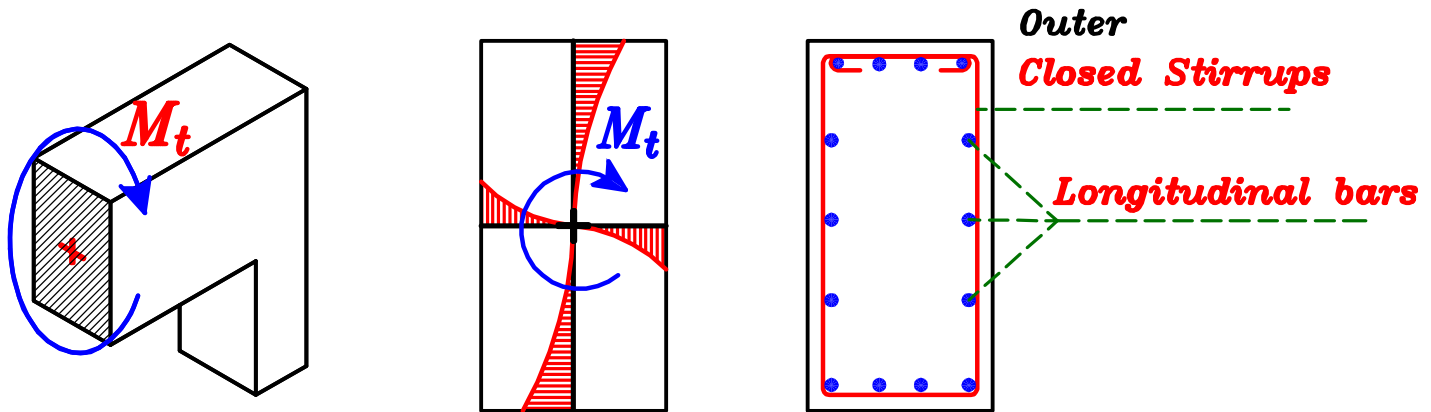
افضل تسليح لمقاومه الشرخ
هو الحديد العمودى على الشرخ
و لكن تنفيذه بهذا الشكل غير عملى



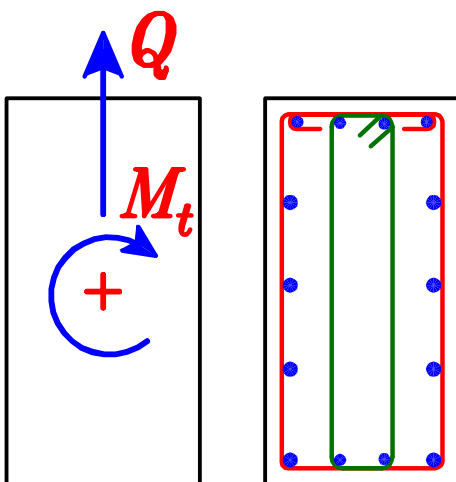
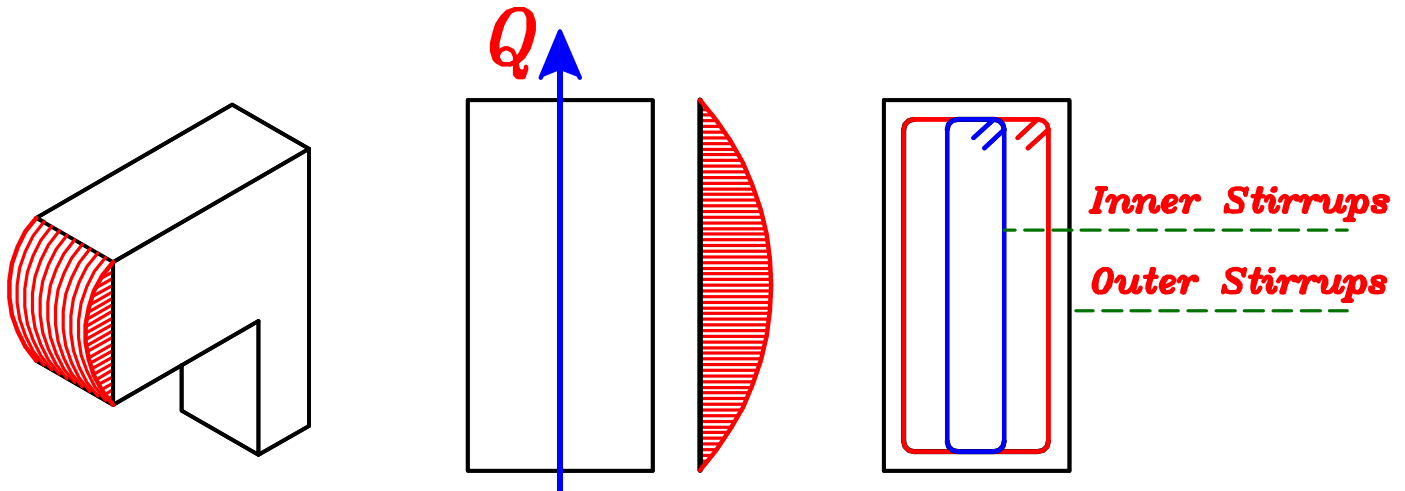
لذا عمليا يوضع تسليح افقى و راسى
لمقاومه الشرخ و هى كانات راسيه
و اسياخ افقيه بطول الكمره

Shear + Torsion RFT.

لمقاومه ال **Torsional moment** منفردا
نحتاج لكانات خارجيه مغلقه **Closed Stirrups**
و نحتاج لاسياخ طولييه موضوعه على المحيط الخارجى للكمرة **Longitudinal bars**
و لن نحتاج لكانات داخلية لانها لن تقاوم **Torsion** لانها ليست عند اكبر **Stress**



لمقاومه ال **Shear Force** منفردا
نحتاج لكانات خارجيه **Outer Stirrups** و ممكن وضع كانات داخلية **Inner Stirrups**



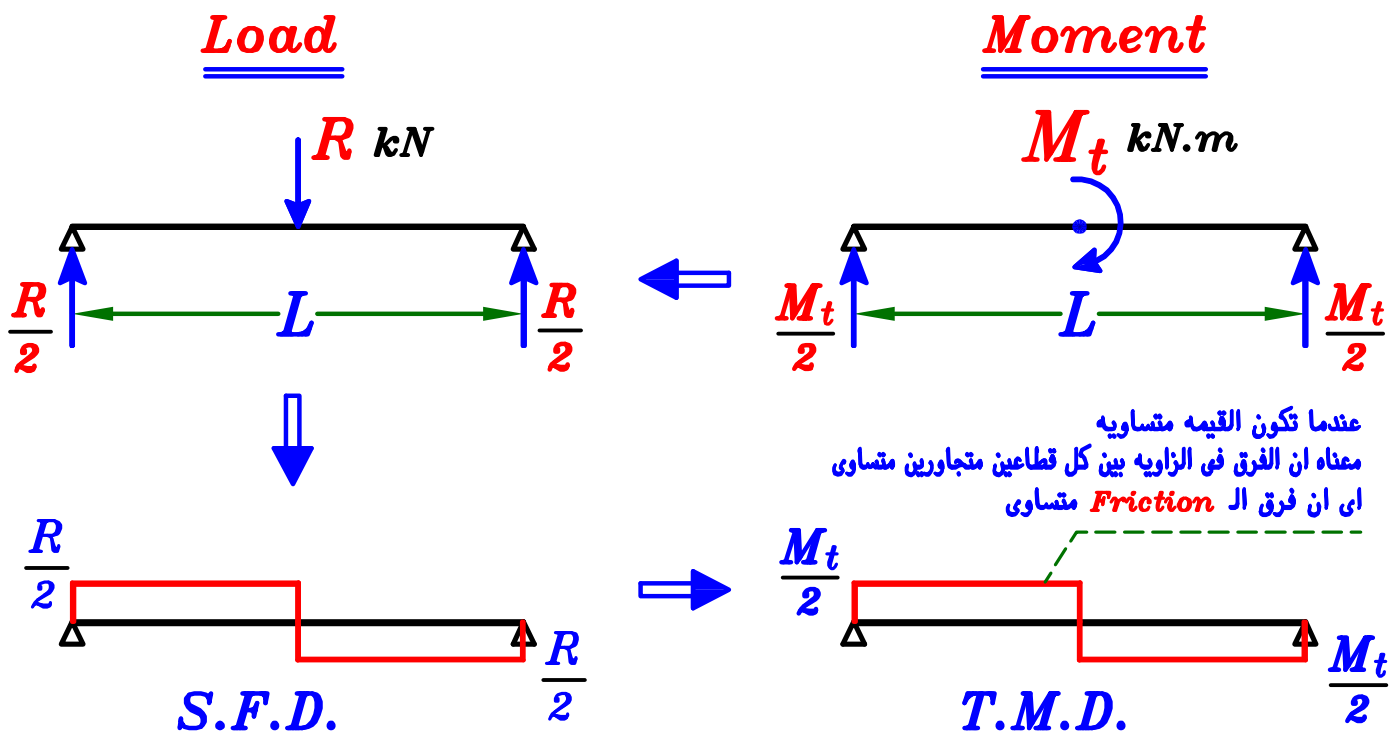
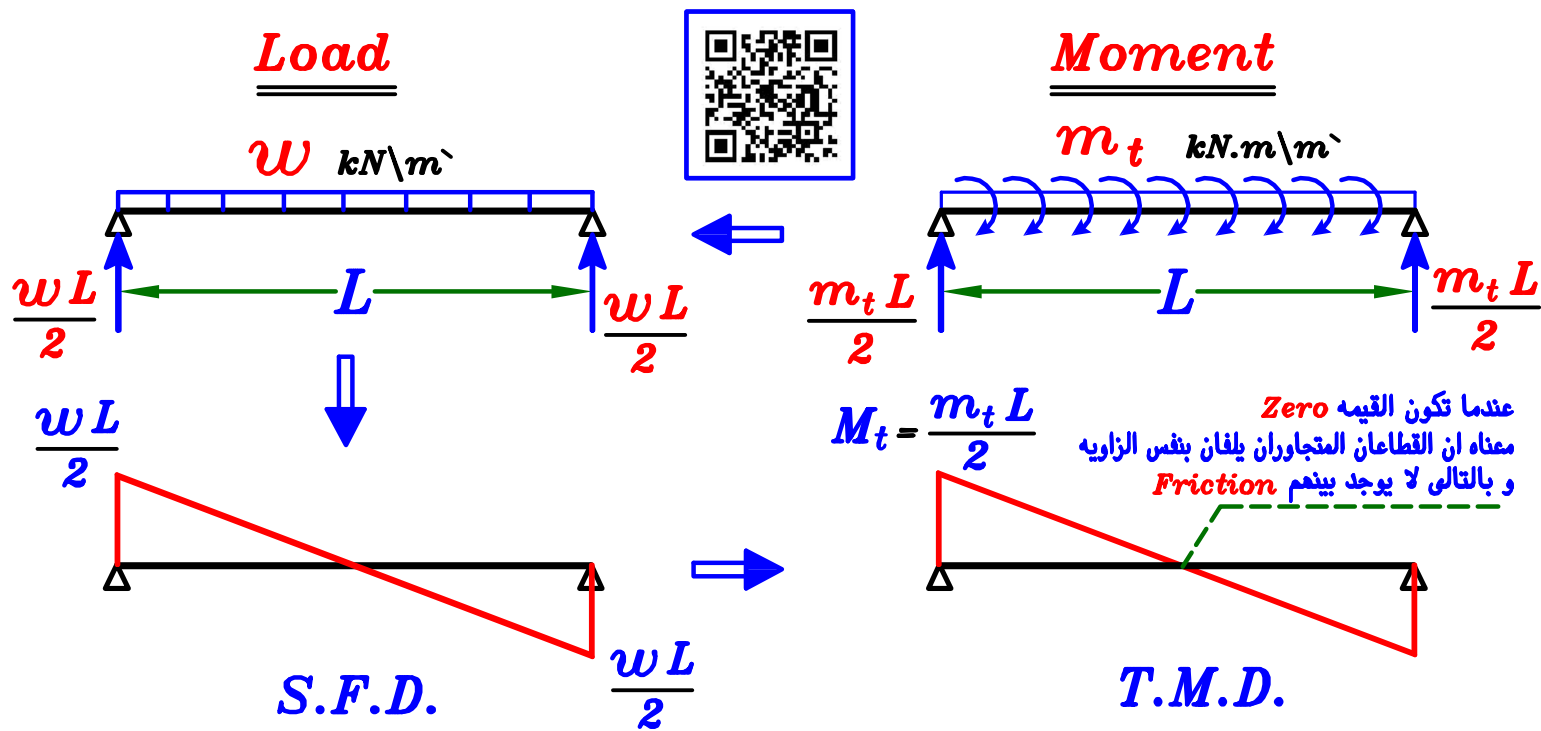
- اذا لمقاومه ال **Shear Force + Torsional moment**
- نضع كانات خارجيه مغلقه **Outer Closed Stirrups** لمقاومه ال **Shear** و ال **Torsion** معاً.
 - ممكن وضع كانات داخلية **Inner Stirrups** لمقاومه ال **Shear** فقط.
 - اسياخ طولييه **Longitudinal bars** لمقاومه ال **Torsion** فقط.

How to Draw Torsional Moment Diagram (T.M.D.)

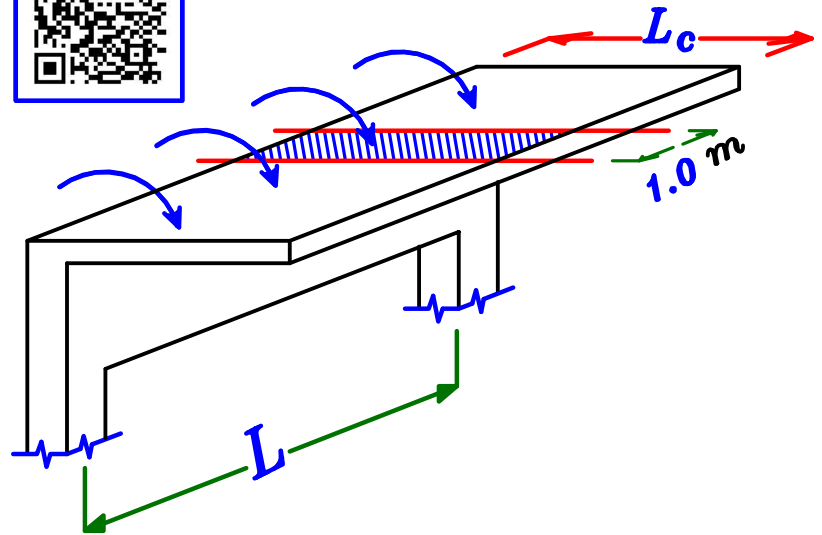
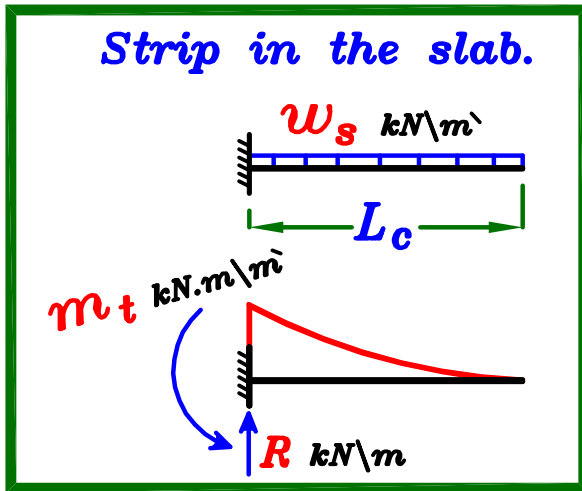
لرسم ال *Torsional Moment Diagram (T.M.D.)* يرسم بنفس طريقة رسم ال *S.F.D.*

لذا عند رسم ال *T.M.D.* نفرض أن ال *Moment* عبارة عن *Load* و نرسم

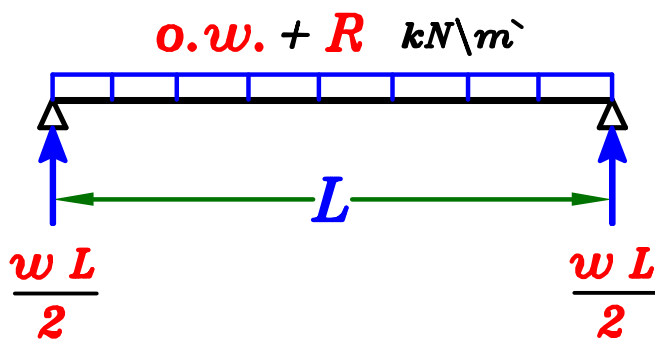
ال *S.F.D.* لهذا ال *Load* فيكون هو نفس شكل ال *T.M.D.* لل *Moment*



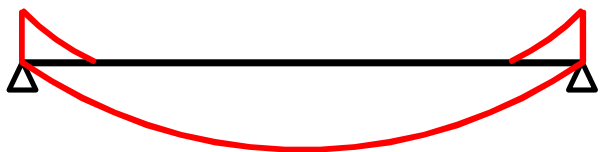
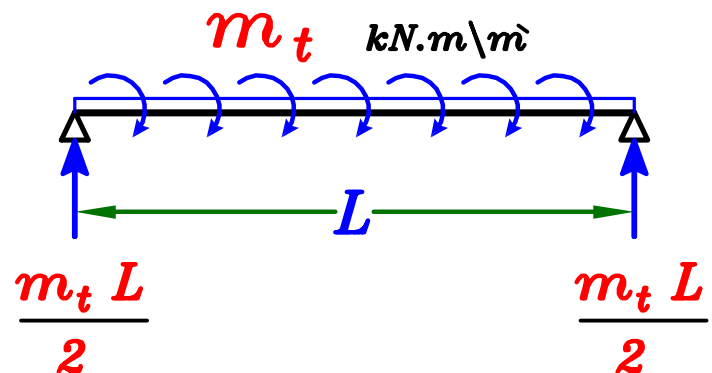
Example.



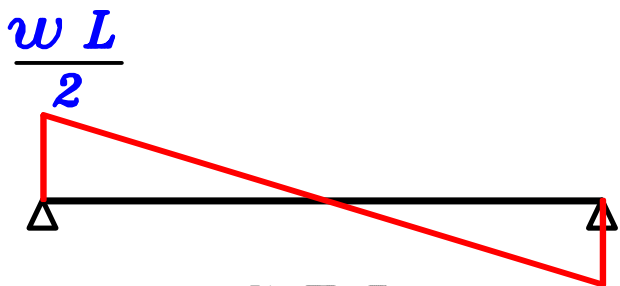
Load



Moment

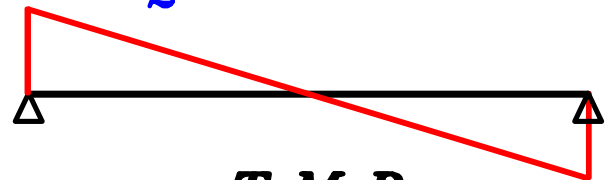


B.M.D.



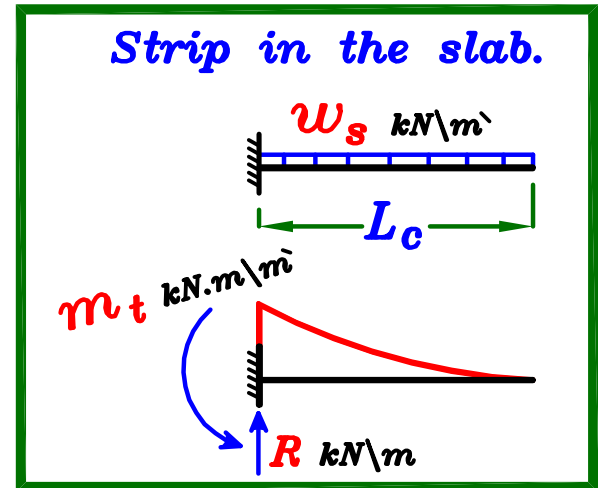
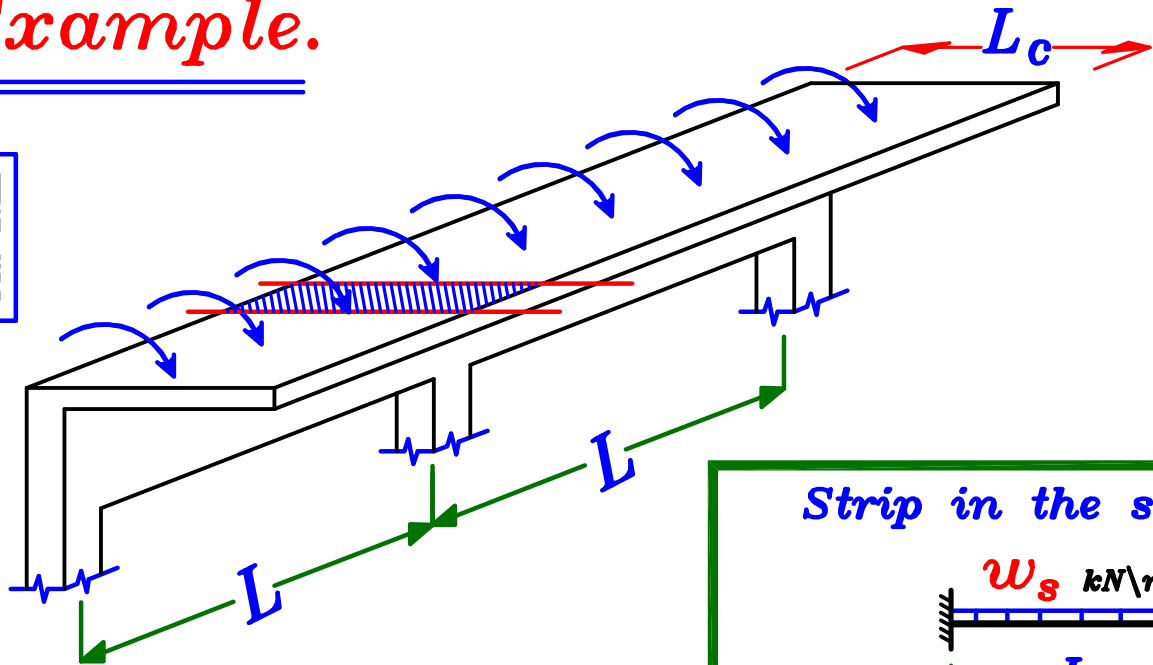
S.F.D.

$$M_t = \frac{m_t L}{2}$$



T.M.D.

Example.

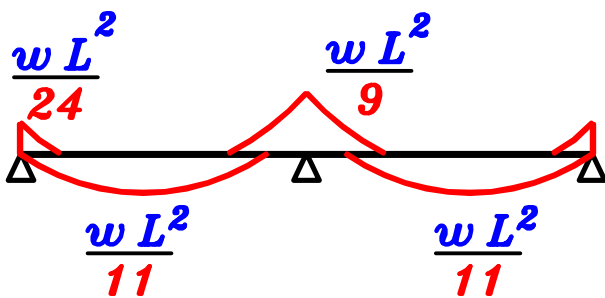
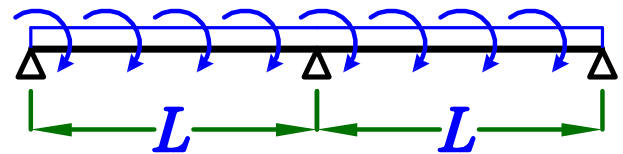
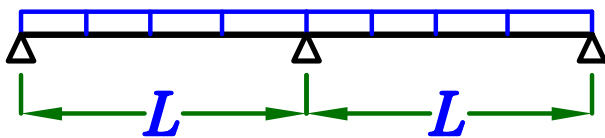


Load

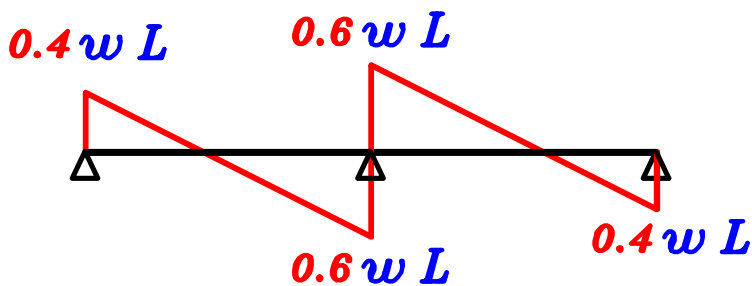
Moment

$$w = o.w. + R \text{ kN/m}$$

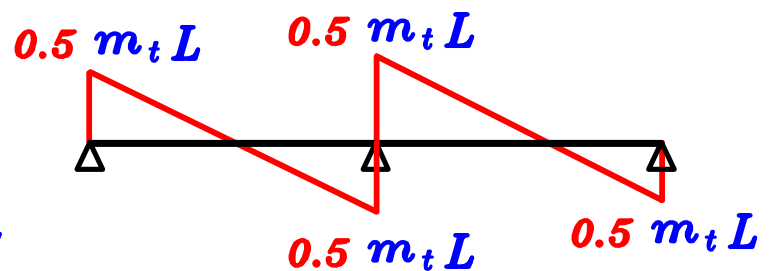
$$m_t \text{ kN.m/m}$$



B.M.D.

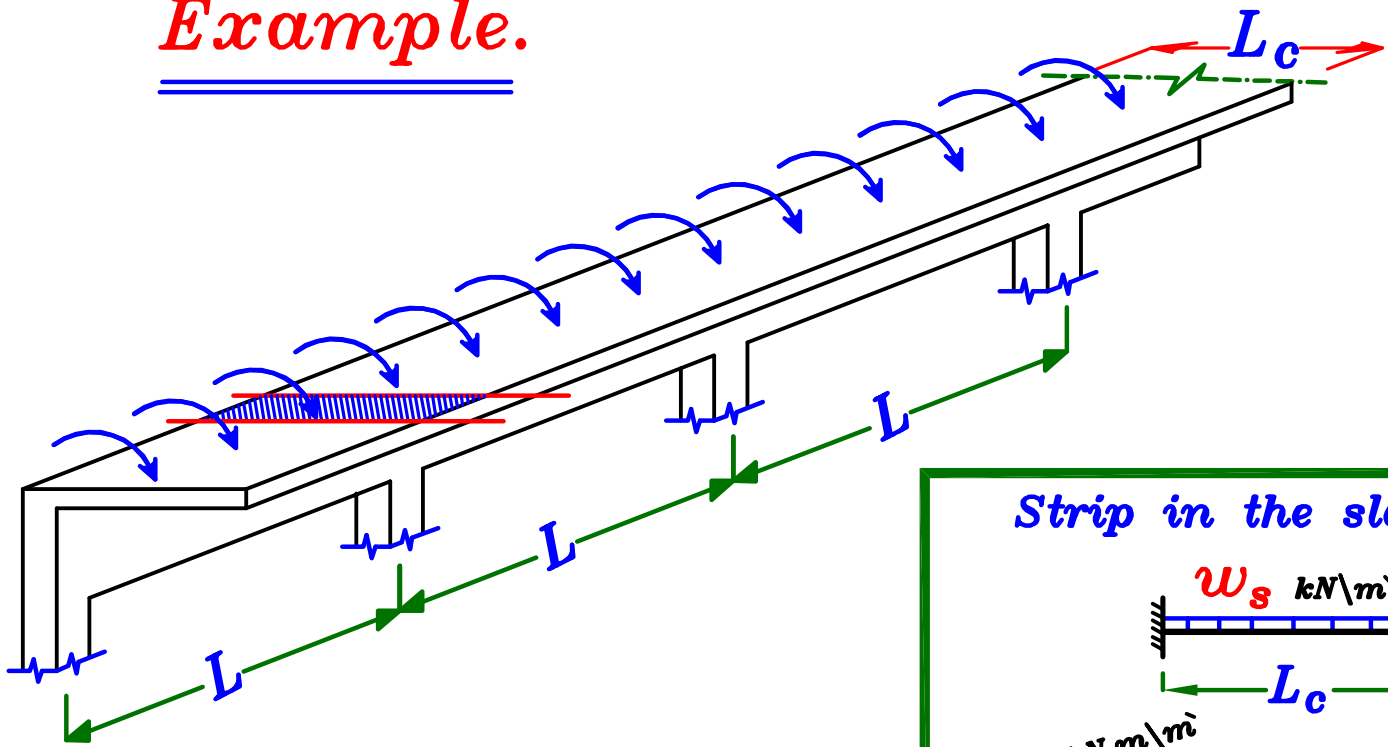


S.F.D.

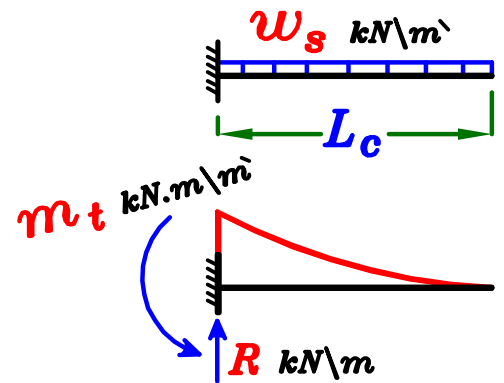


T.M.D.

Example.



Strip in the slab.

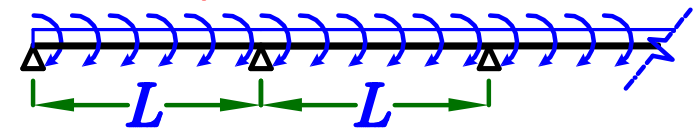
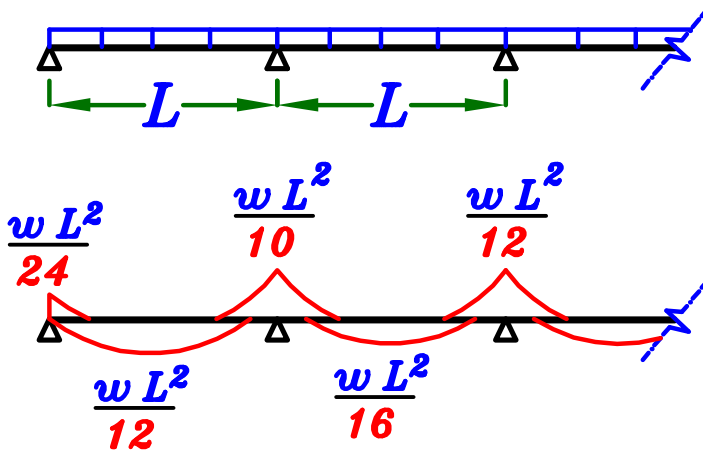


Load

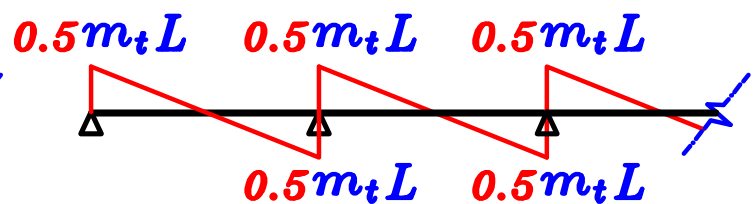
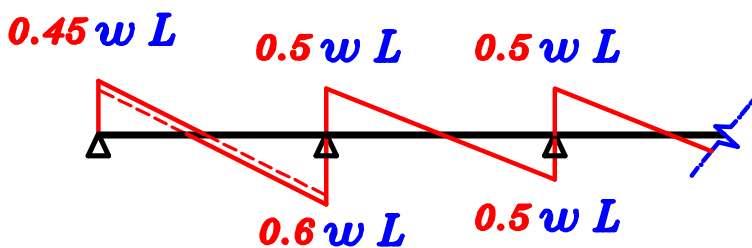
$$W = 0.w. + R \quad kN/m$$

Moment

$$m_t \quad kN.m/m$$



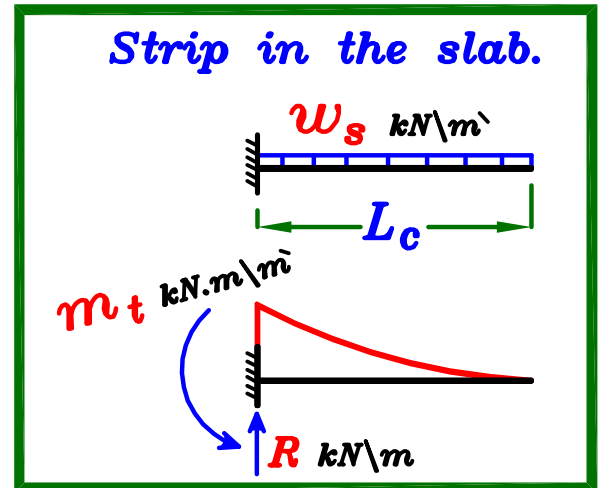
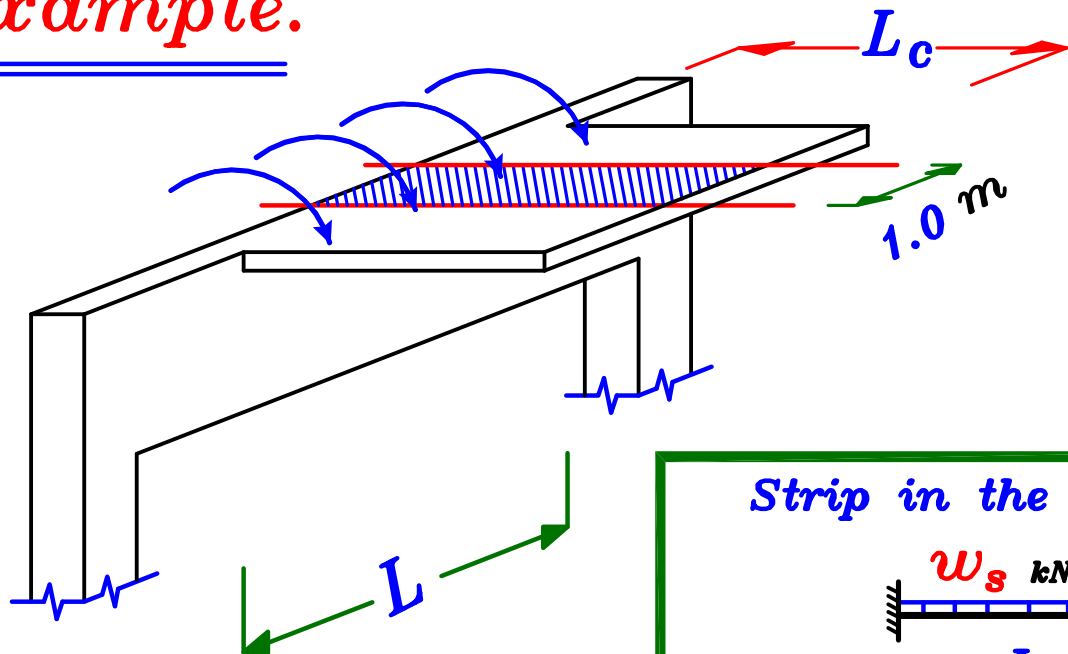
B.M.D.



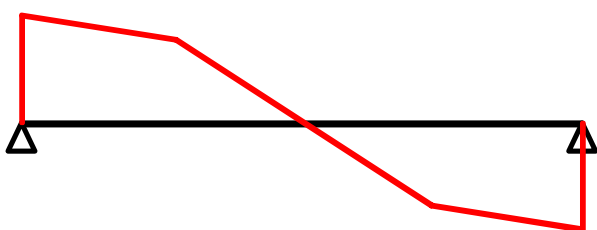
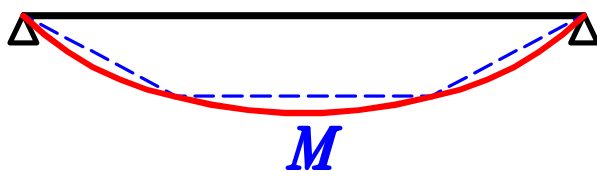
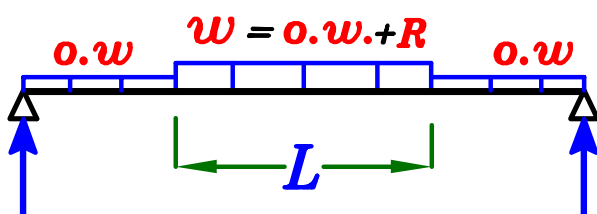
S.F.D.

T.M.D.

Example.

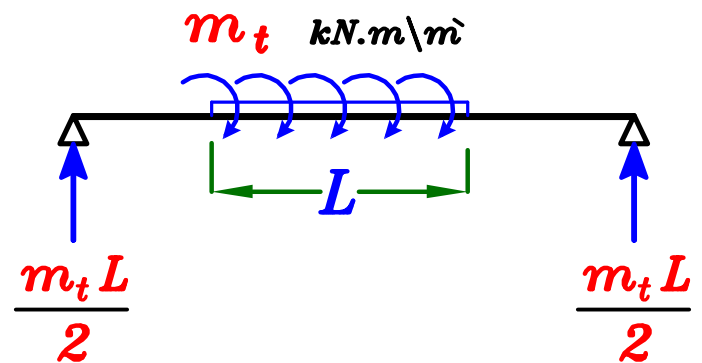


Load



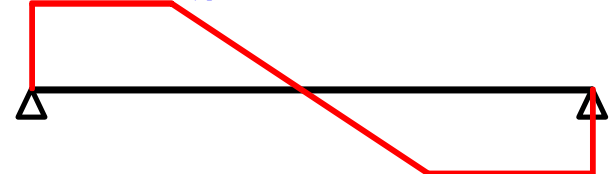
S.F.D.

Moment



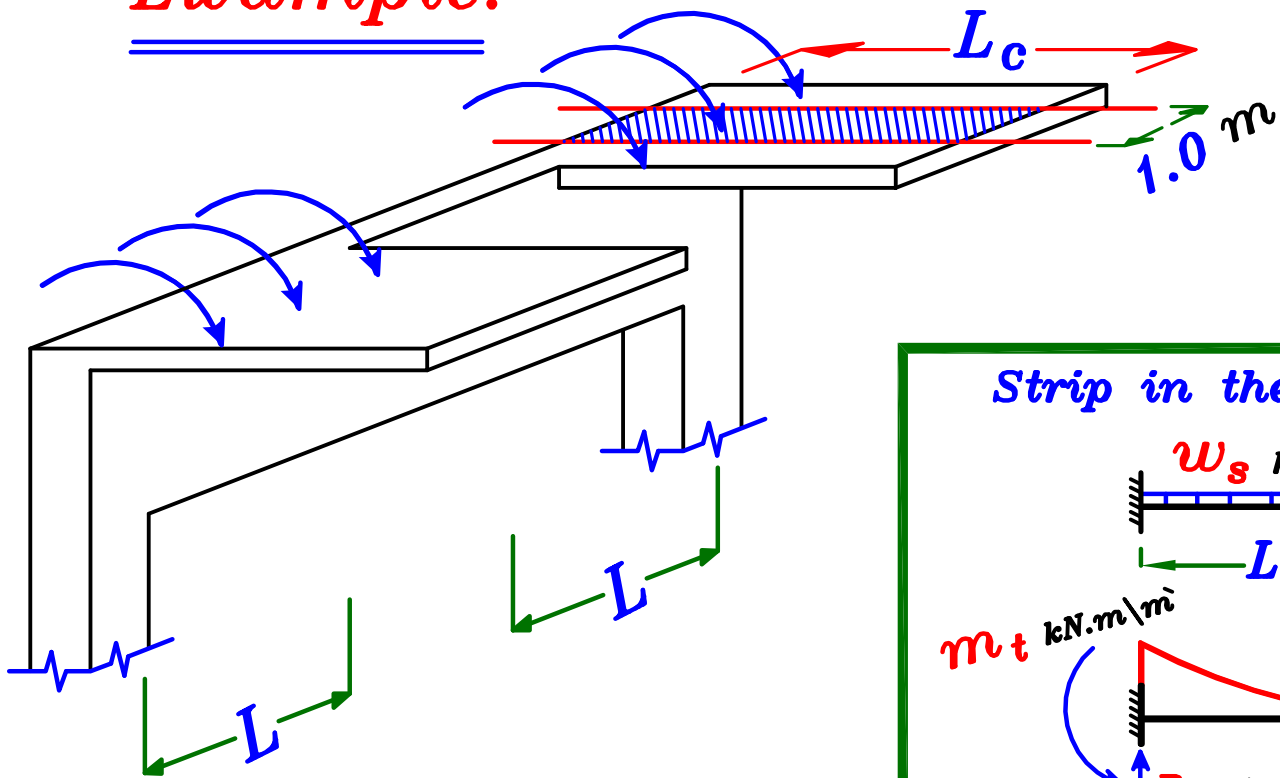
B.M.D.

$$M_t = \frac{m_t L}{2}$$

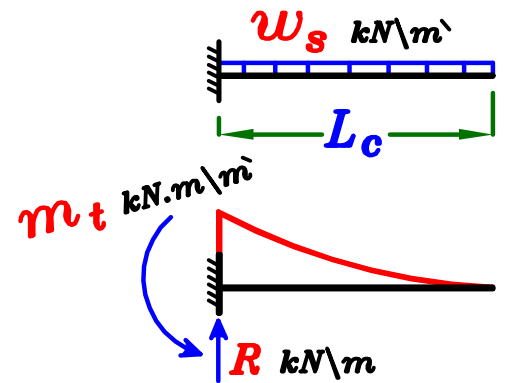


T.M.D.

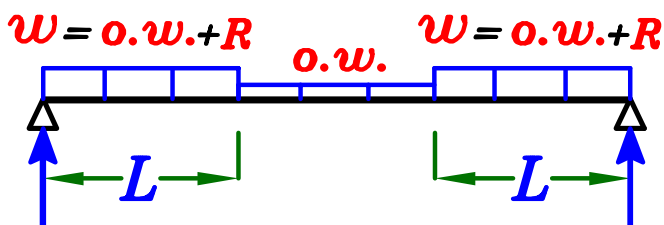
Example.



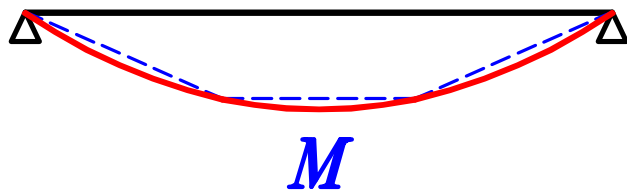
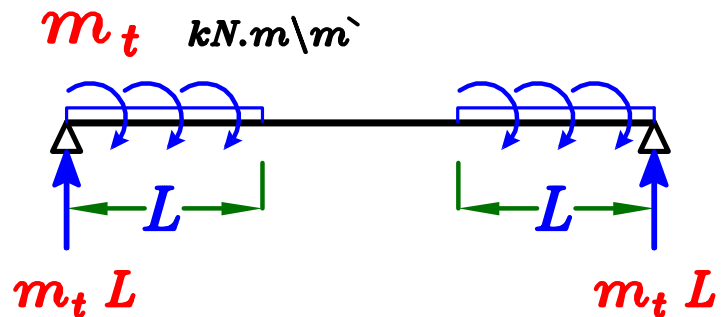
Strip in the slab.



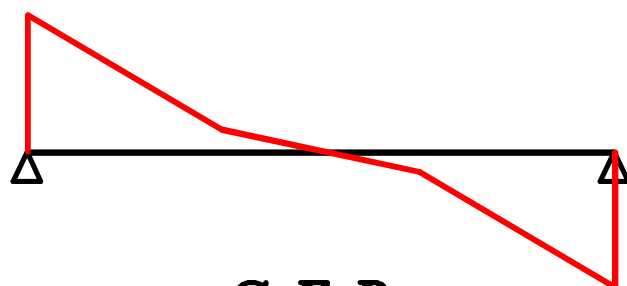
Load



Moment

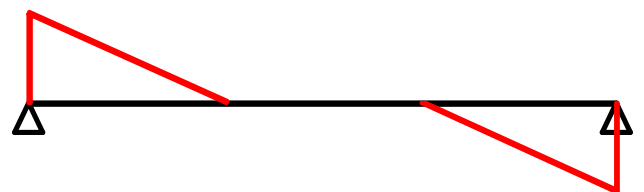


B.M.D.



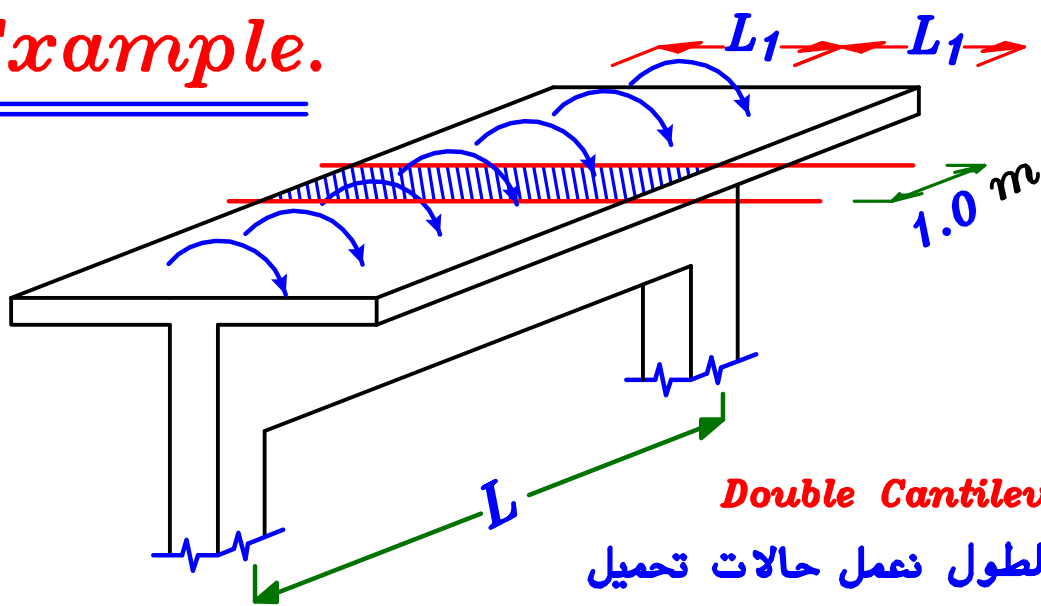
S.F.D.

$$M_t = m_t L$$



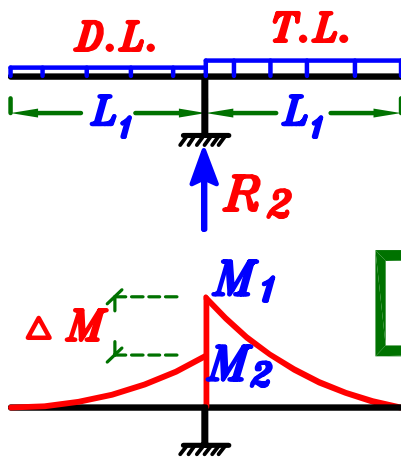
T.M.D.

Example.

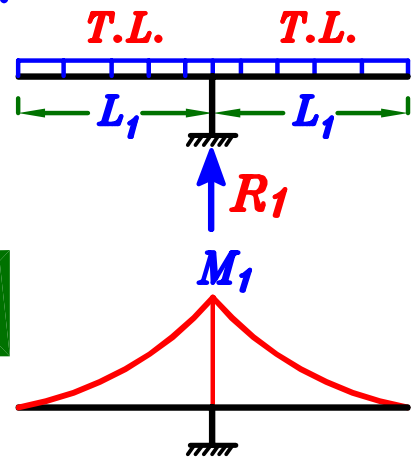


عند وجود **Double Cantilever** متساويه فى الطول نعمل حالات تحميل فنضع **T.L.** على أى منهما

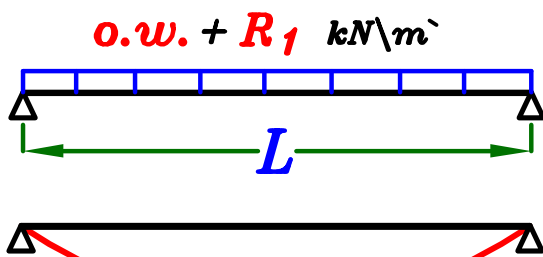
Strip in the slab.



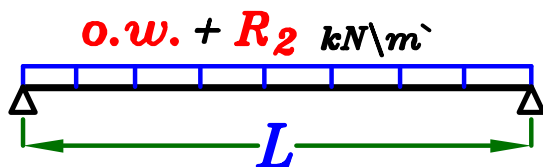
$$\Delta M = M_1 - M_2$$



Load

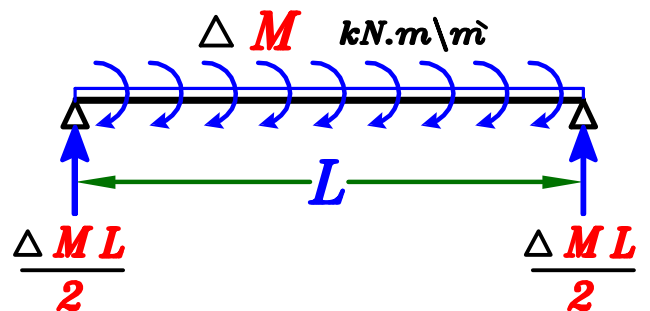


B.M.D.



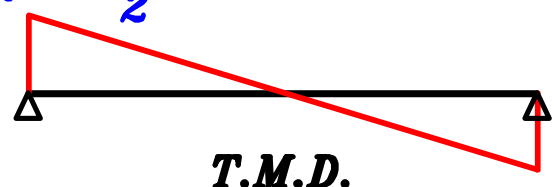
S.F.D.

Moment



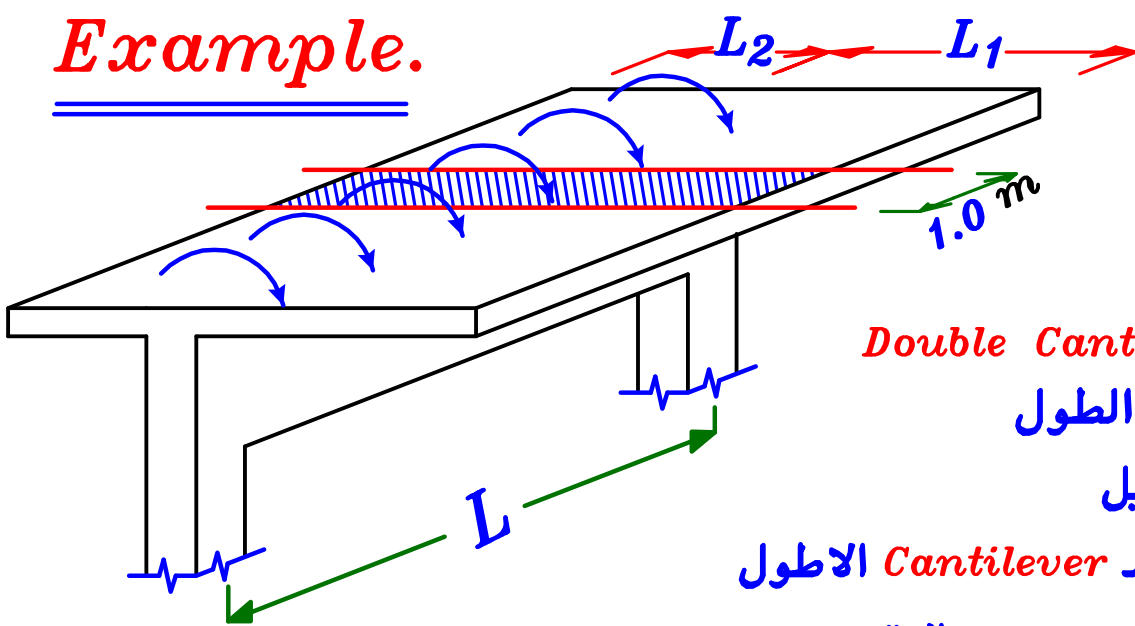
فى التصميم يجب التصميم على ال **Shear** و ال **Torsion** من نفس حاله التحميل

$$M_t = \frac{\Delta M L}{2}$$

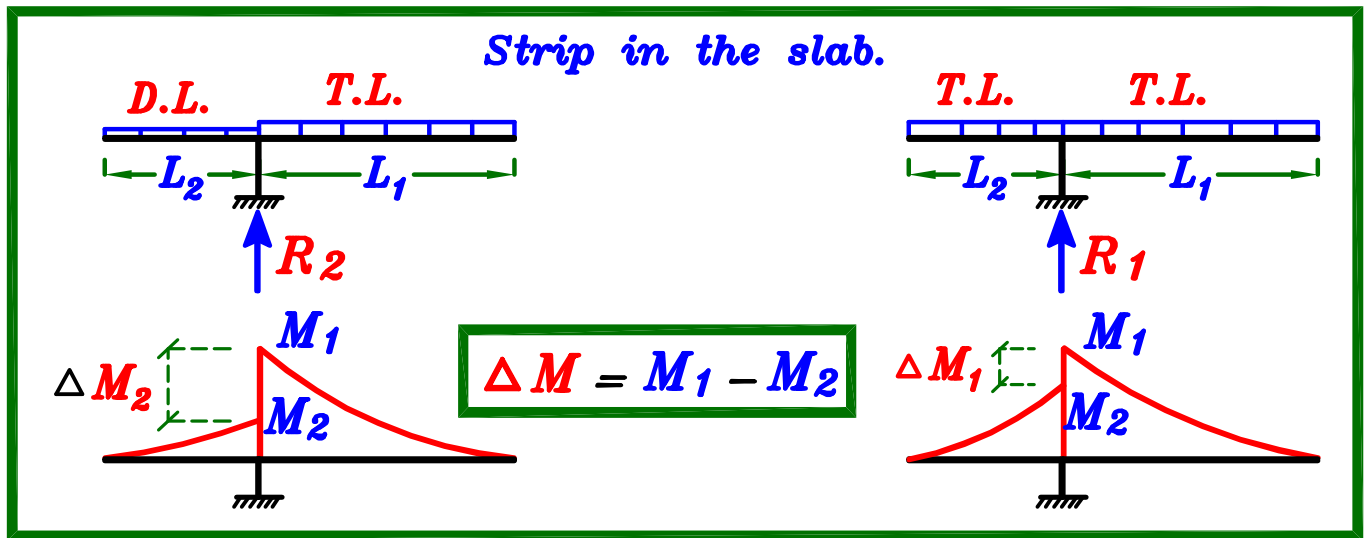


T.M.D.

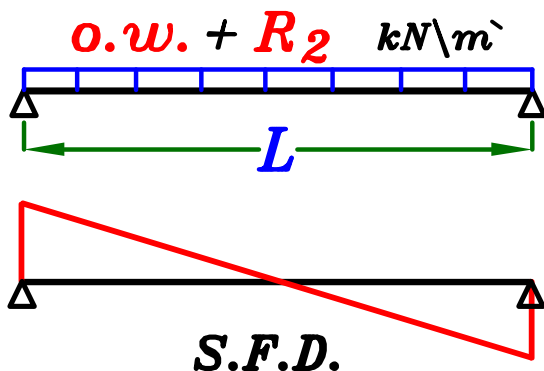
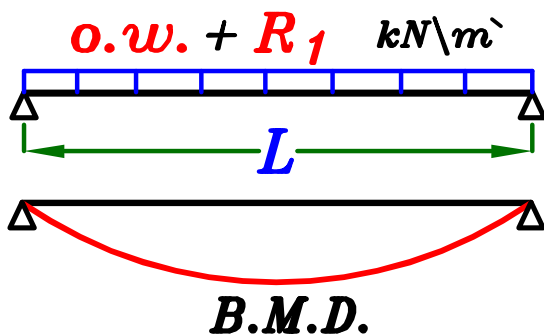
Example.



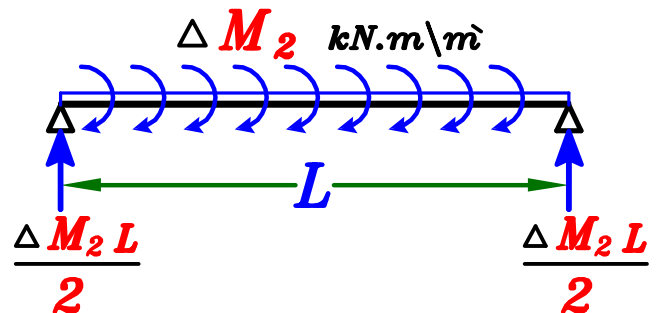
عند وجود **Double Cantilever** غير متساويه فى الطول
نعمل حالات تحميل
فنضع **T.L.** على ال **Cantilever** الاطول
و **D.L.** على ال **Cantilever** الاقصر



Load

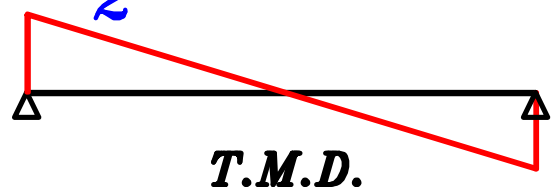


Moment

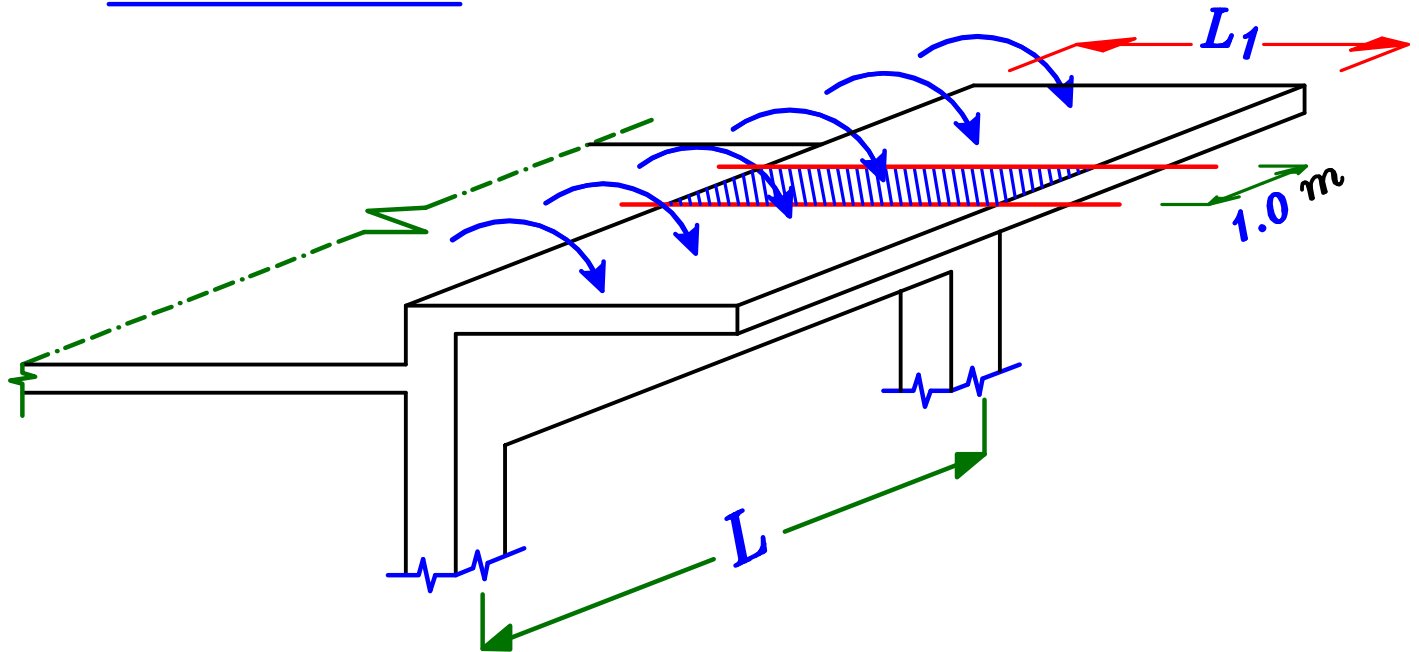


فى التصميم يجب التصميم على
ال **Shear** و ال **Torsion** من نفس حاله التحميل

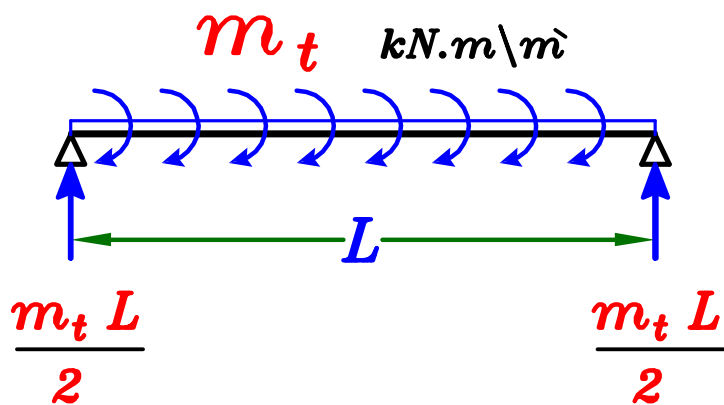
$$M_t = \frac{\Delta M_2 L}{2}$$



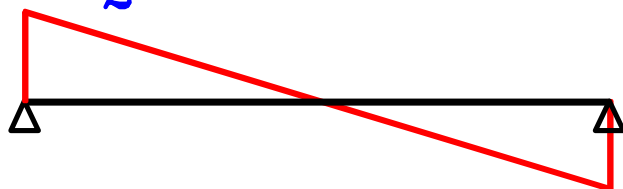
Example.



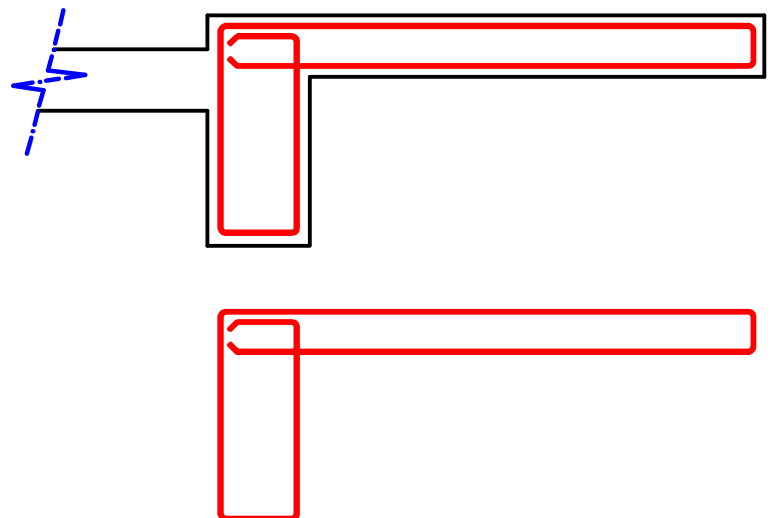
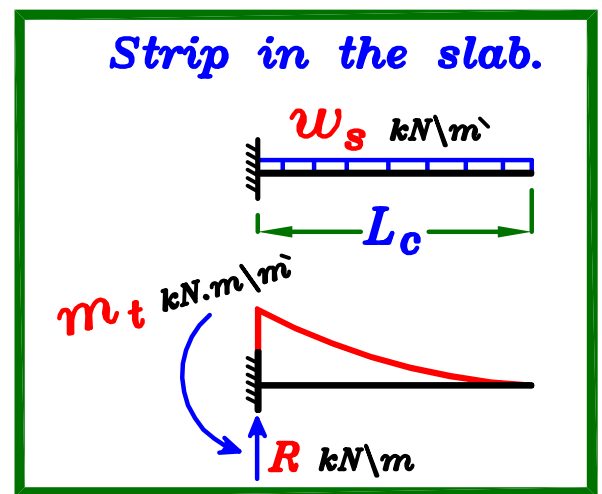
Moment



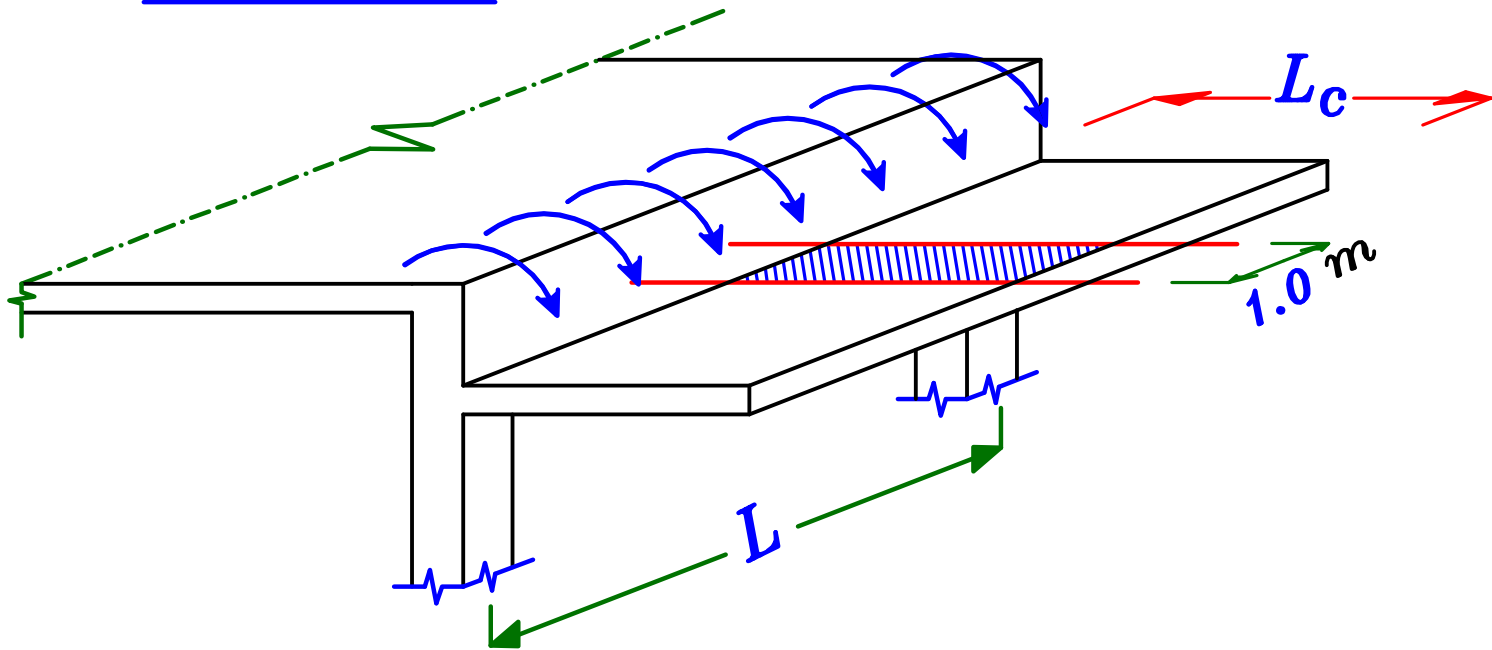
$$M_t = \frac{m_t L}{2}$$



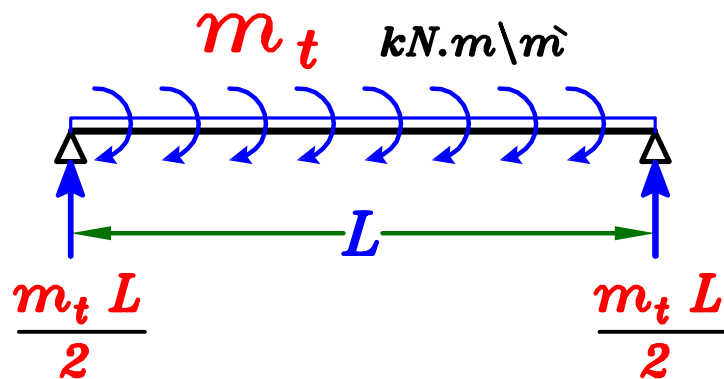
T.M.D.



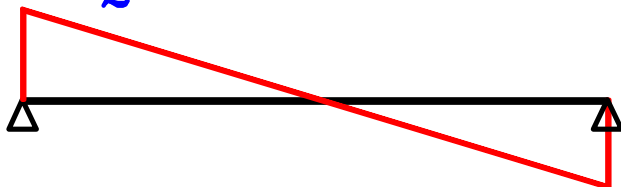
Example.



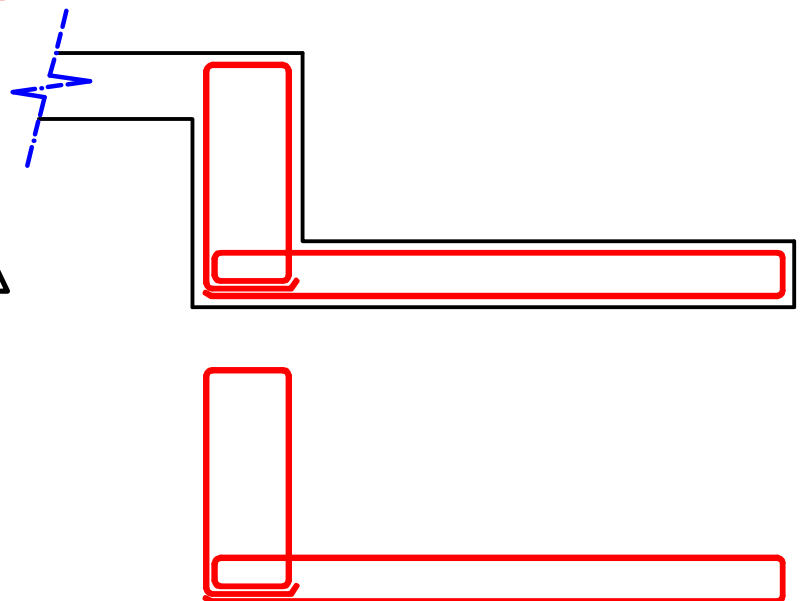
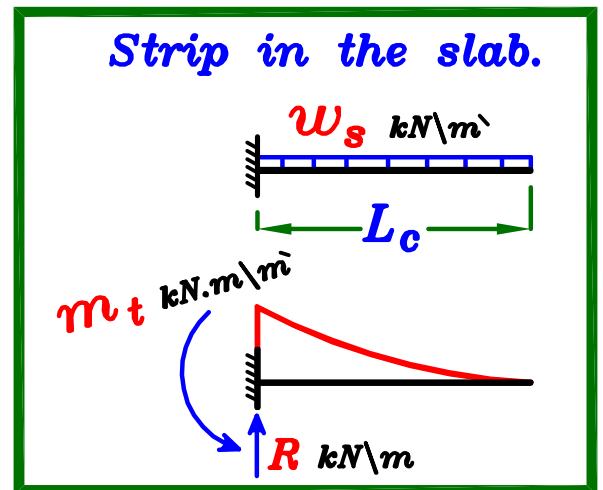
Moment



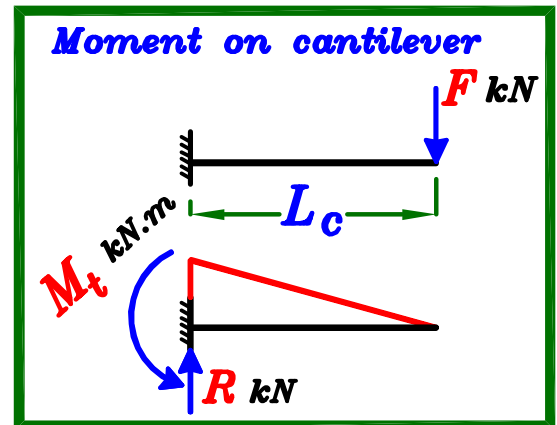
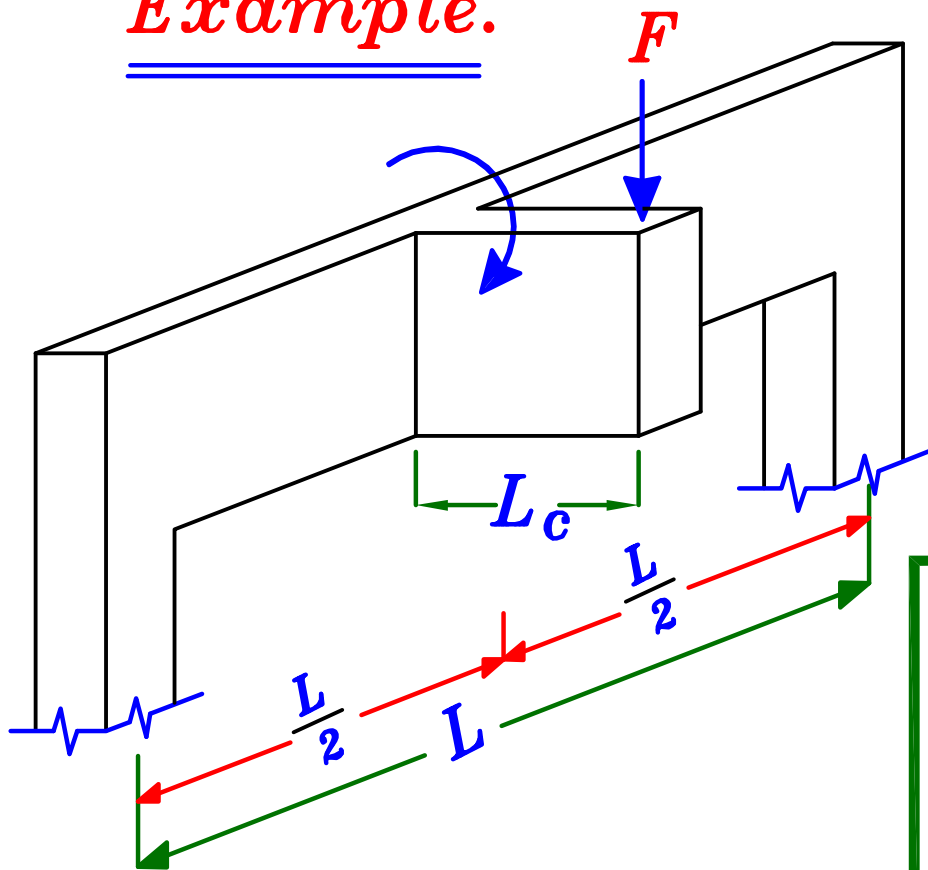
$$M_t = \frac{m_t L}{2}$$



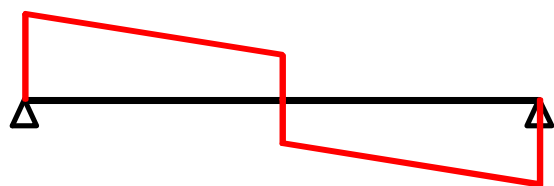
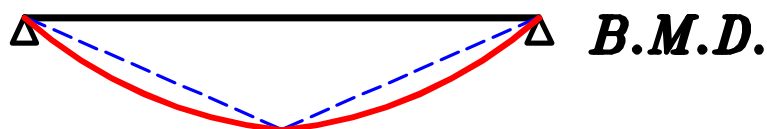
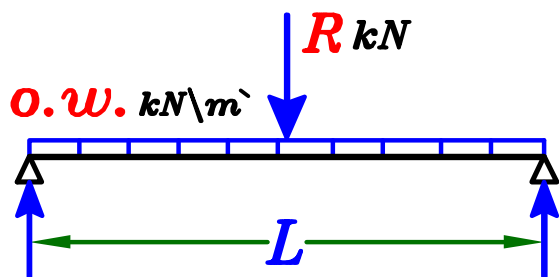
T.M.D.



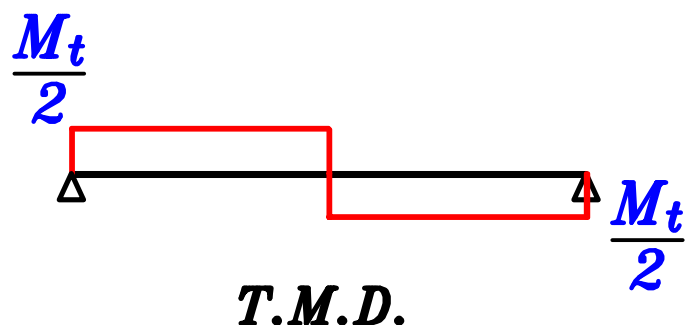
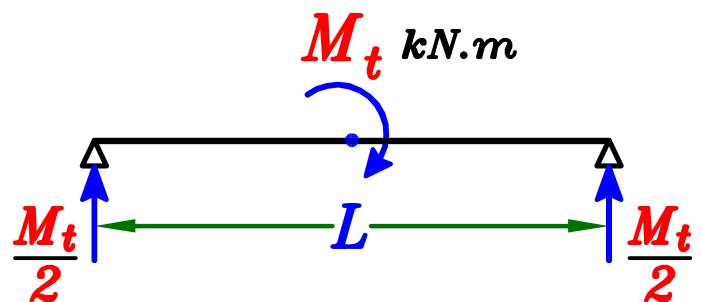
Example.



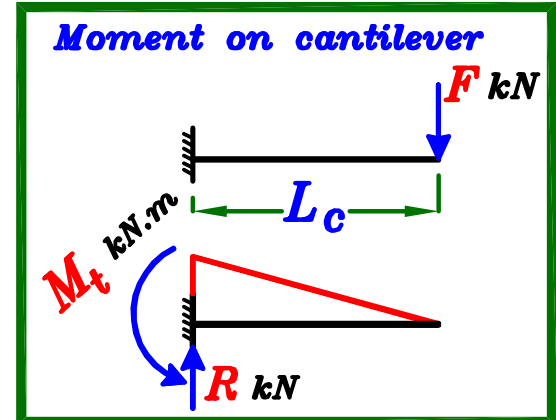
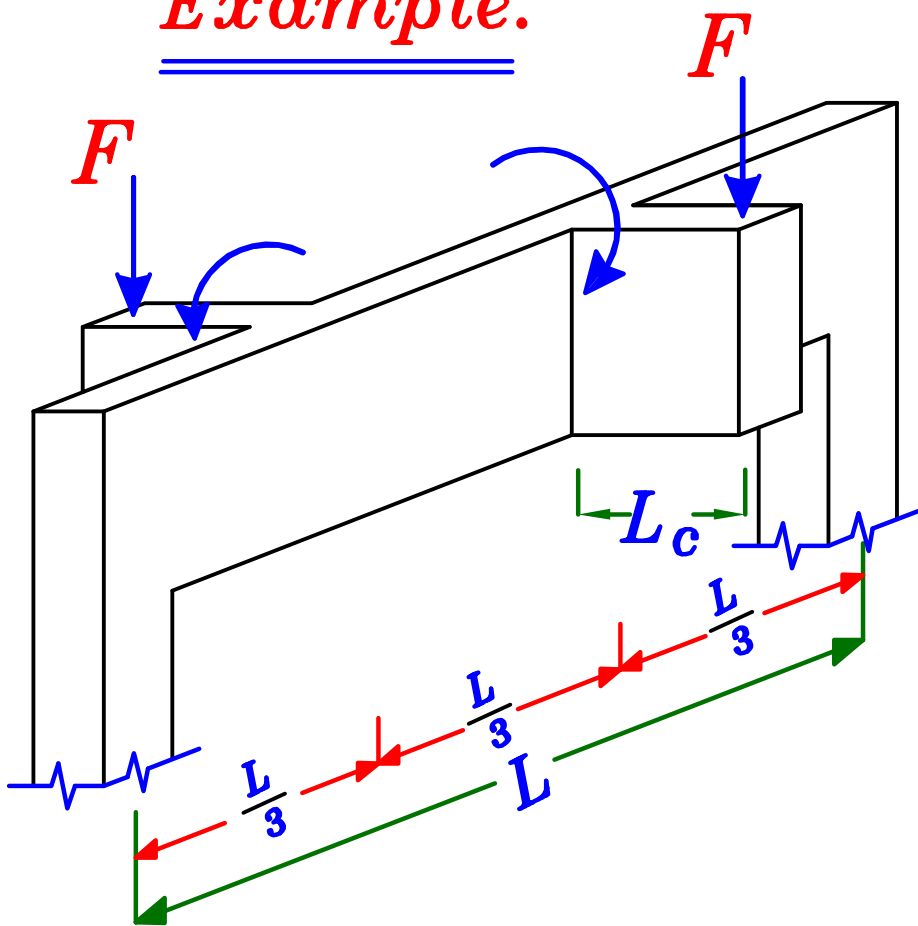
Load



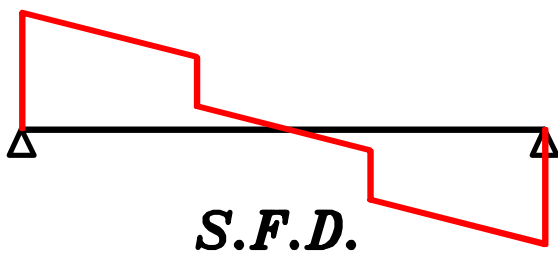
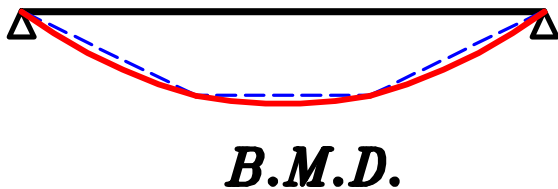
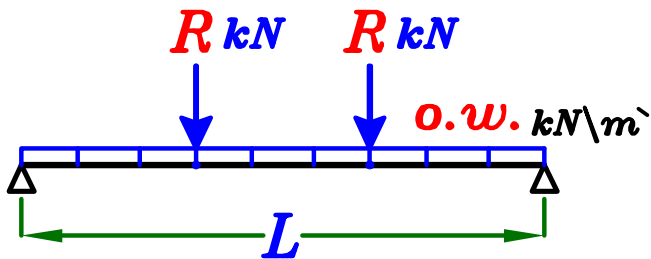
Moment



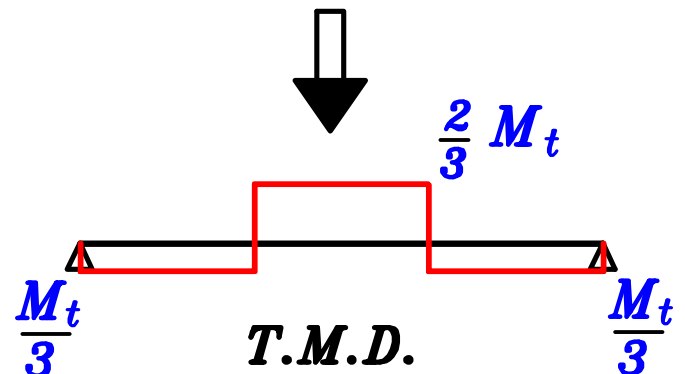
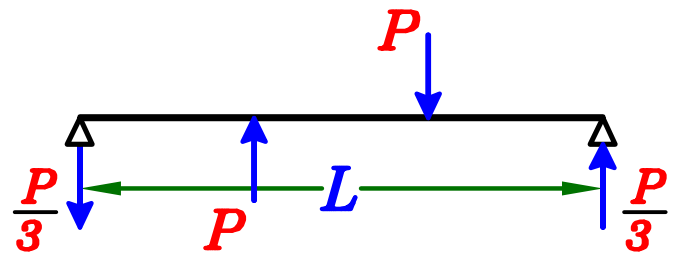
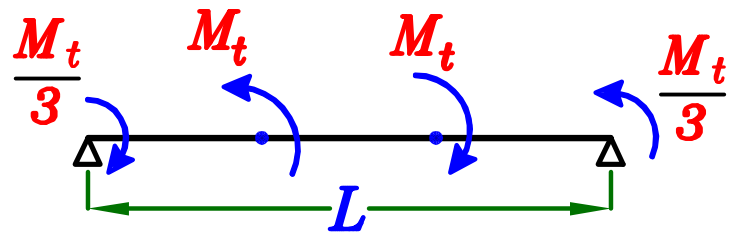
Example.



Load

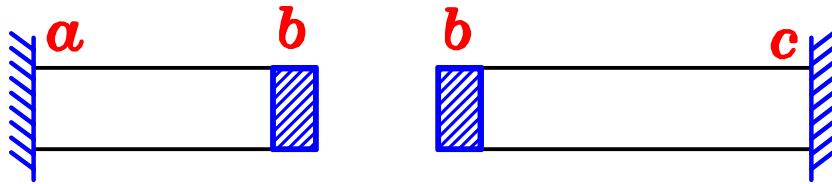
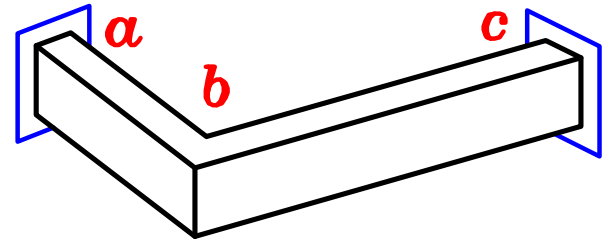
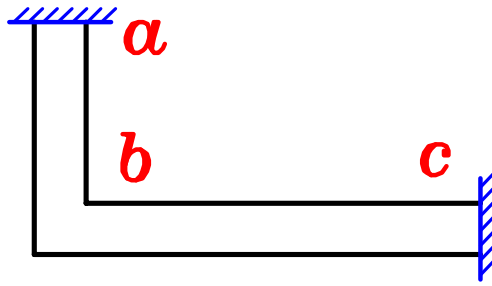


Moment

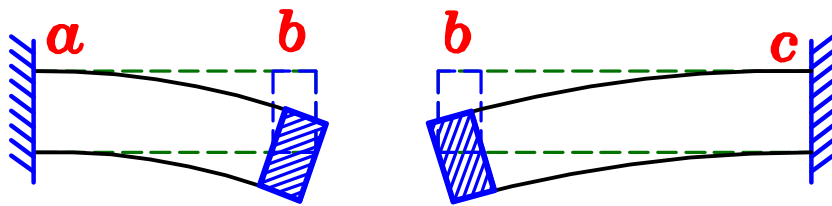
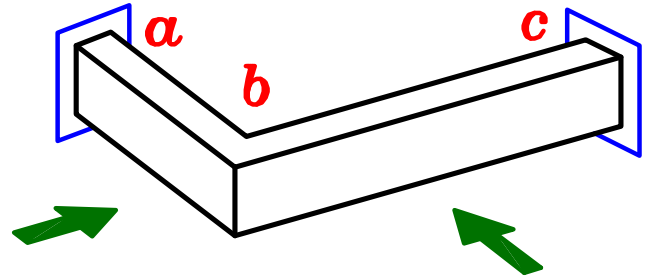




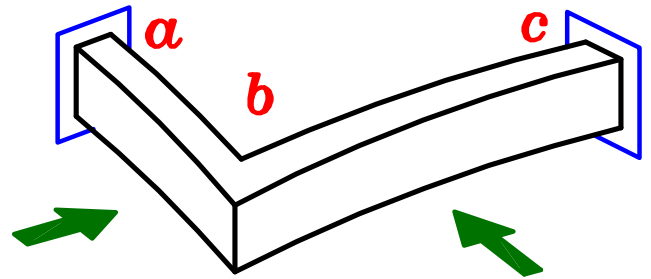
Plan



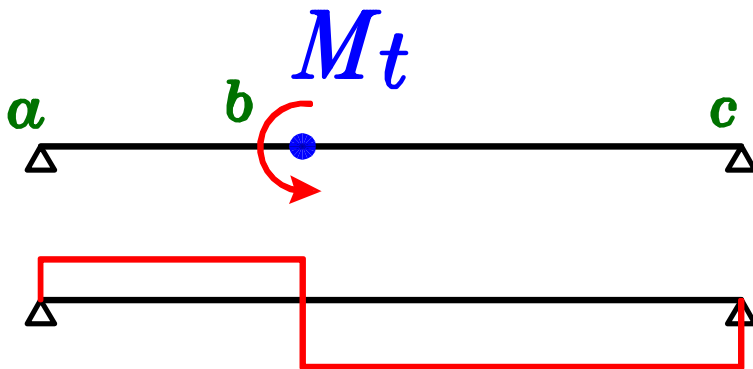
Before Deflection



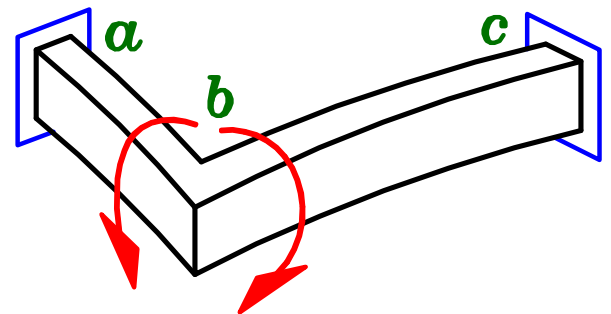
After Deflection



الكمرات التي بها كسرات أفقيه يكون عليها **Concentrated torsional Moment** **Deflection** لانه يحدث انحناء لكل جزء من الكمره عند حدوث ال
 مما يعمل على دوران القطاع للجزء الاخر من الكمره .
 و يكون اتجاه ال **Torsion** للخارج

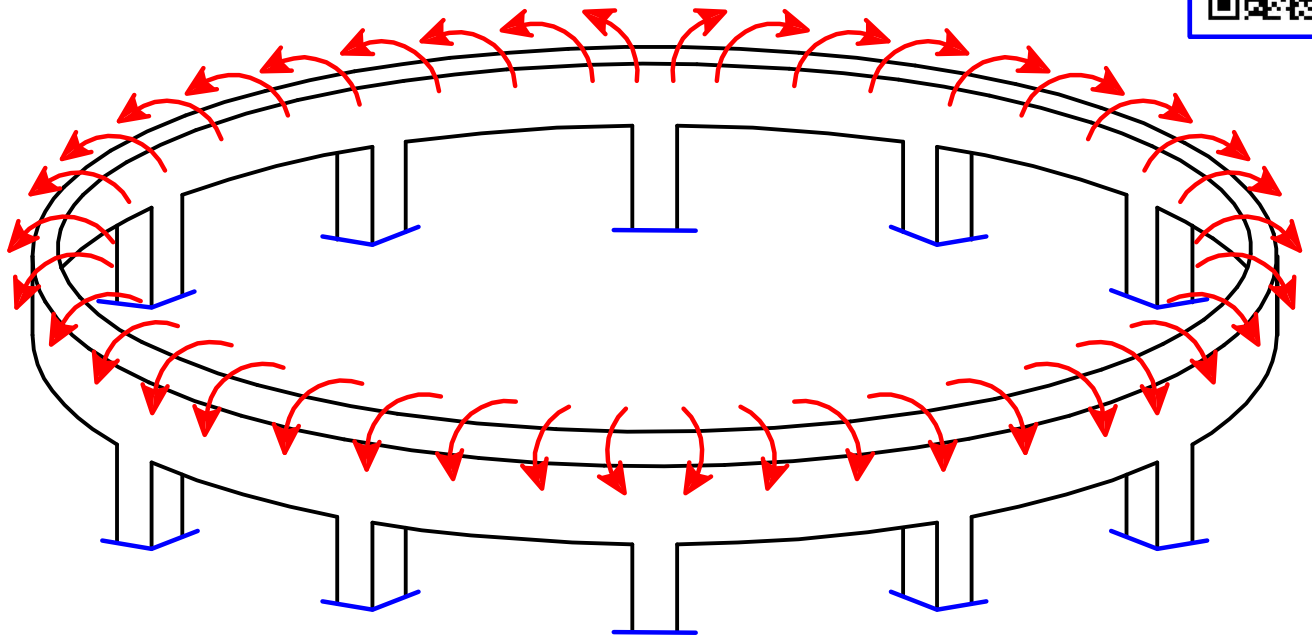


T.M.D.



لا توجد معادلات يدويه لحساب قيمه هذا ال **Torsion**
 لذا عند حساب قيمته نحتاج لاستخدام برامج الكمبيوتر .

Circular Beams.



الكمرات الدائرية مثل الكمرات التي بها كسرات أفقية و لكن ال **Torsion** فيها **Distributed torsional Moment**

لحساب ال **Bending Moment & Shear Force & Torsional Moment**

Old Tables Page 120

المؤثرين على الكمره ممكن استخدام الجدول التالى

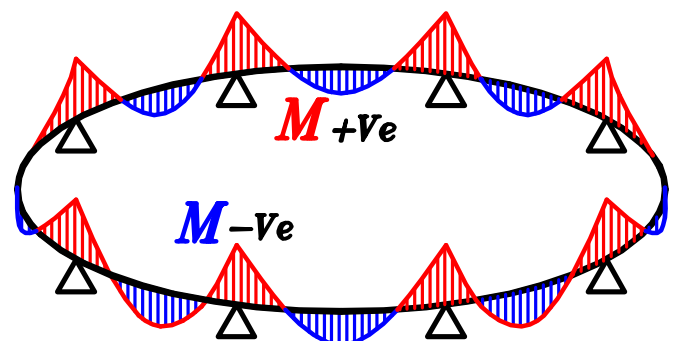
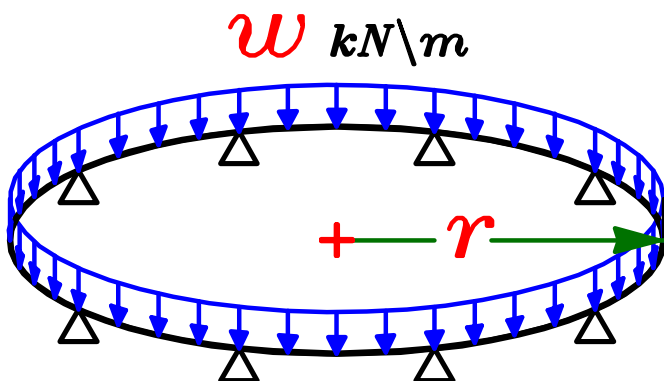
$$P = w * 2\pi r$$

P = Total load on the beam. (kN)

w = Load per meter. (kN/m)

r = Radius of the beam. (m)

n = Number of supports.



Data for Design of Reinforced Concrete Structures

1. Circular Beams

Supported on a number of supports (n) at equal distance under uniformly dist'd load (pt/m')

$$M = pr^2 \left[\pi/n \frac{\cos \phi}{\sin \phi} - 1 \right]$$

$$M_t = -pr^2 \left[\pi/n \frac{\sin \phi}{\sin \phi} - \phi \right]$$

$$Q = -p.r.\phi$$

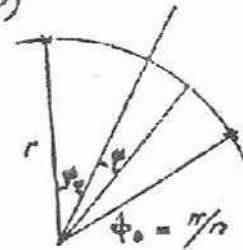


Table of extreme values
 $P = 2\pi r.p$

Number of supports (n)	Load on each column R	Max. Shearing force Q_{max}	Max. Bending M		Max. Torsional moment M_t	Central angle between Axis of supports Θ
			At center of span $M (+)$	Over Support $M (-ve)$		
4	$P/4$	$P/8$	$0.0176 Pr$	$-0.0322 Pr$	$0.0053 Pr$	$19^\circ 21'$
6	$P/6$	$P/12$	$0.0075 Pr$	$-0.0148 Pr$	$0.0015 Pr$	$12^\circ 44'$
8	$P/8$	$P/16$	$0.0042 Pr$	$-0.0083 Pr$	$0.0006 Pr$	$9^\circ 33'$
10	$P/10$	$P/20$	$0.0032 Pr$	$-0.0052 Pr$	$0.0004 Pr$	$7^\circ 36'$
12	$P/12$	$P/24$	$0.0019 Pr$	$-0.0037 Pr$	$0.0002 Pr$	$6^\circ 21'$

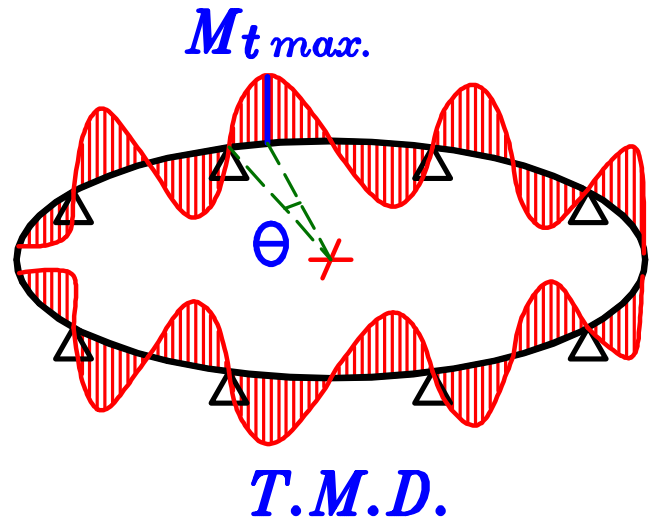
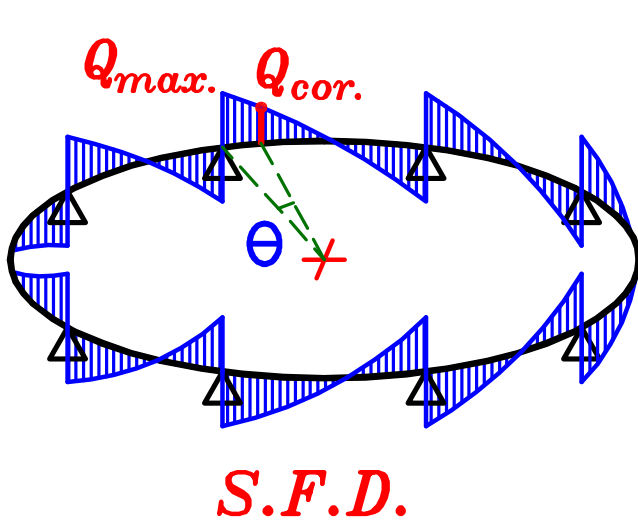
No. of supports	Load on each support	Max. Shearing Force	Max. Bending Moment		Max. Torsional Moment	Central angle
			at C.L. of Span	Over C.L. of Column		
n	R	Q_{max}	$M +ve$	$M -ve$	$M_{t max}$	Θ
4	$P/4$	$P/8$	$0.0176 Pr$	$-0.0322 Pr$	$0.0053 Pr$	$19^\circ 21'$
6	$P/6$	$P/12$	$0.0075 Pr$	$-0.0148 Pr$	$0.0015 Pr$	$12^\circ 44'$
8	$P/8$	$P/16$	$0.0042 Pr$	$-0.0083 Pr$	$0.0006 Pr$	$9^\circ 33'$
10	$P/10$	$P/20$	$0.0032 Pr$	$-0.0052 Pr$	$0.0004 Pr$	$7^\circ 36'$
12	$P/12$	$P/24$	$0.0019 Pr$	$-0.0037 Pr$	$0.0002 Pr$	$6^\circ 21'$

ال Θ Central angle هي الزاوية المقاسة من ال Support حتى النقطة التي يوجد عندها max. Torsional moment

١- ال (θ) Central angel هي الزاوية المقاسه من ال **Support** حتى النقطة التي يوجد عندها **max. Torsional moment**

أى أن ال **Section** الذى يوجد عنده **max. Torsional moment** مكانه غير ال **Section** الذى يوجد عنده **max. Shear Force**

لذا عند تصميم الكانات لتحمل **Shear + Torsion** نحدد قيمه **Q corresponding** وهى قيمه ال **Shear Force** عند ال **Section** الذى يوجد عنده **max. Torsion**

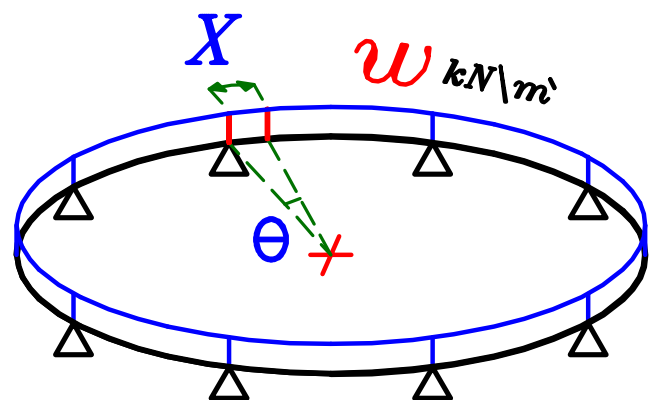


Radian

$$X = r * \theta = r * \theta * \frac{\pi}{180}$$

$$X = r * \theta * \frac{\pi}{180}$$

$$Q_{cor.} = Q_{max} - w * X$$



يمكن للتسهيل تصميم القطاع على $(M_{t max.} , Q_{max})$

٢- اذا كان عدد ال **Supports** اكبر من او يساوى ١٢ ($n \geq 12$) فمن الممكن :

أ- نعمل عزم الالتواء (M_t) لان قيمته ستكون صغيره جدا .

ب- ممكن حساب ال **max. Bending Moment** & **max. Shear Force** كالآتى :

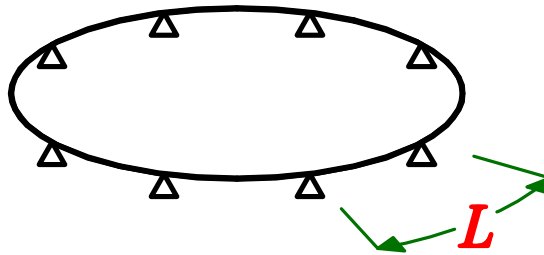
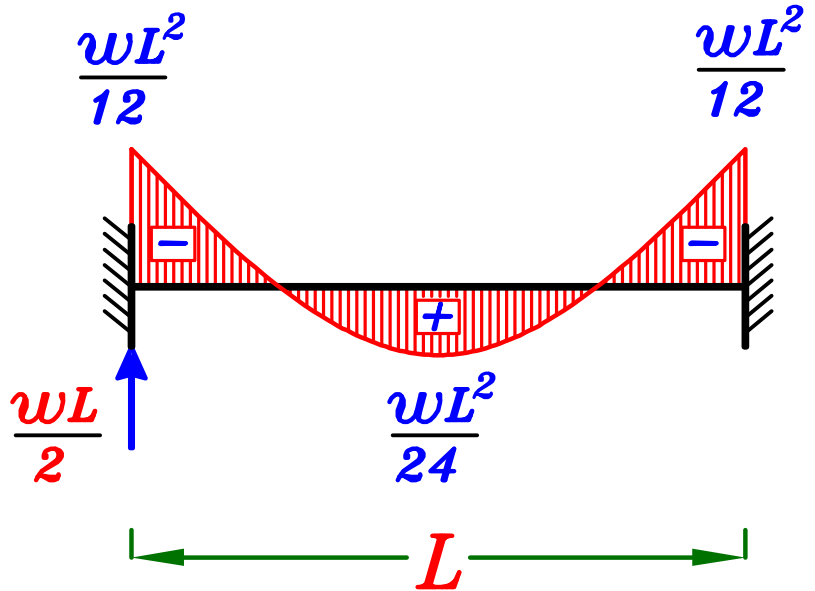
$$\max. M_{-ve} = \frac{wL^2}{12}$$

$$\max. M_{+ve} = \frac{wL^2}{24}$$

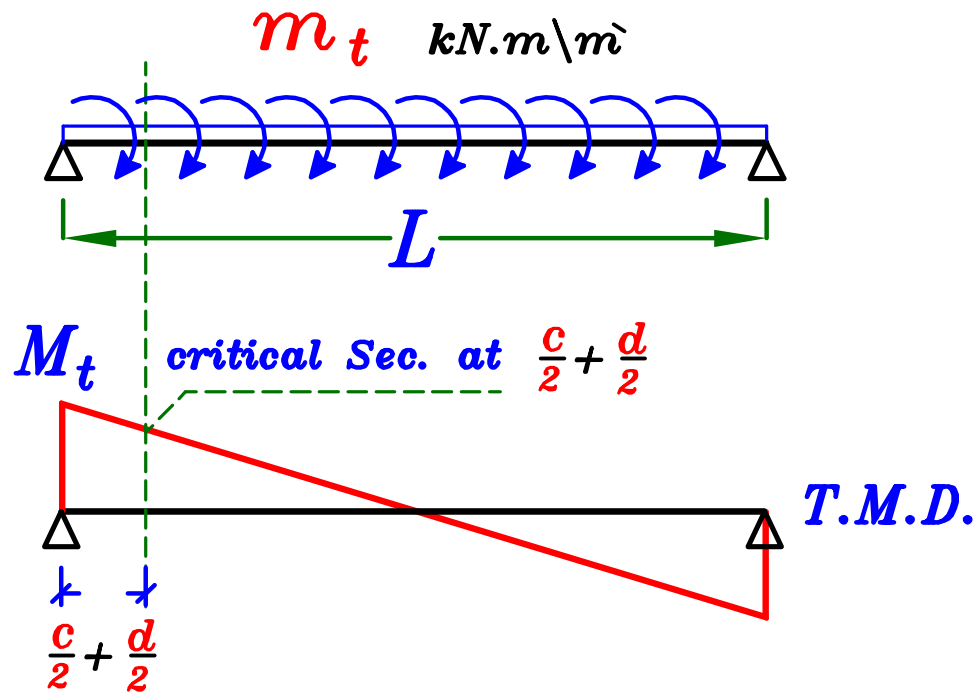
$$Q_{\max.} = \frac{wL}{2}$$

where

$$L = \frac{2\pi r}{n}$$



Shear Stress due to Torsional moment. (q_{tu})



$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} \quad (N/mm^2)$$

Where:

- * q_{tu} (N/mm^2) = Actual Shear Stress due to Torsional Moment.
- * M_{tu} ($N.mm$) = Torsional Moment at Critical Section.
- * A_{oh} (mm^2) = المساحة الداخلية للكانه المقاومه لل **Torsion**
- * A_o (mm^2) = $0.85 * A_{oh}$
- * P_h (mm) = محيط الكانه المقاومه لل **Torsion**
- * t_e (mm) = $\frac{\text{المساحة الداخلية للكانه}}{\text{محيط الكانه}} = \frac{A_{oh}}{P_h}$

$$* y_1 = t - 2 \text{ Cover} \approx t - 80 \text{ mm}$$

$$* x_1 = b - 2 \text{ Cover} \approx b - 80 \text{ mm}$$

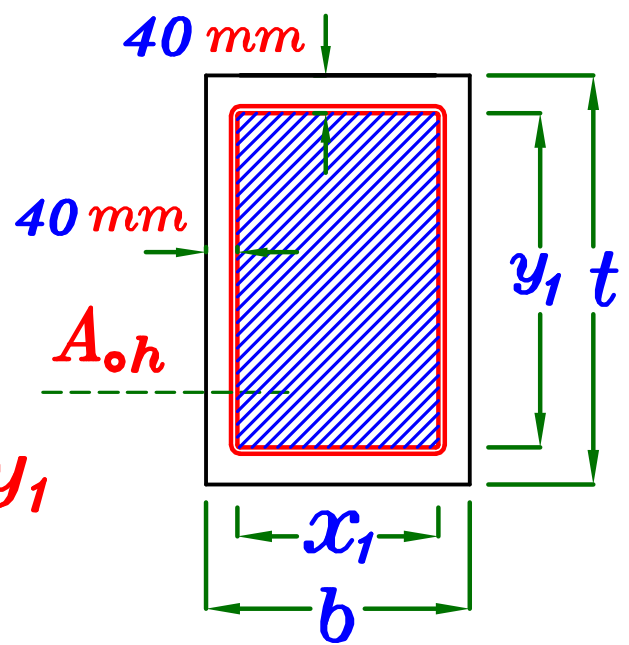
For R-Sec.

$$A_{oh} = \text{المساحة الداخلية للكانه} = x_1 * y_1$$

$$P_h = \text{محيط الكانه} = 2 (x_1 + y_1)$$

$$t_e = \frac{\text{المساحة الداخلية للكانه}}{\text{محيط الكانه}} = \frac{x_1 * y_1}{2 (x_1 + y_1)}$$

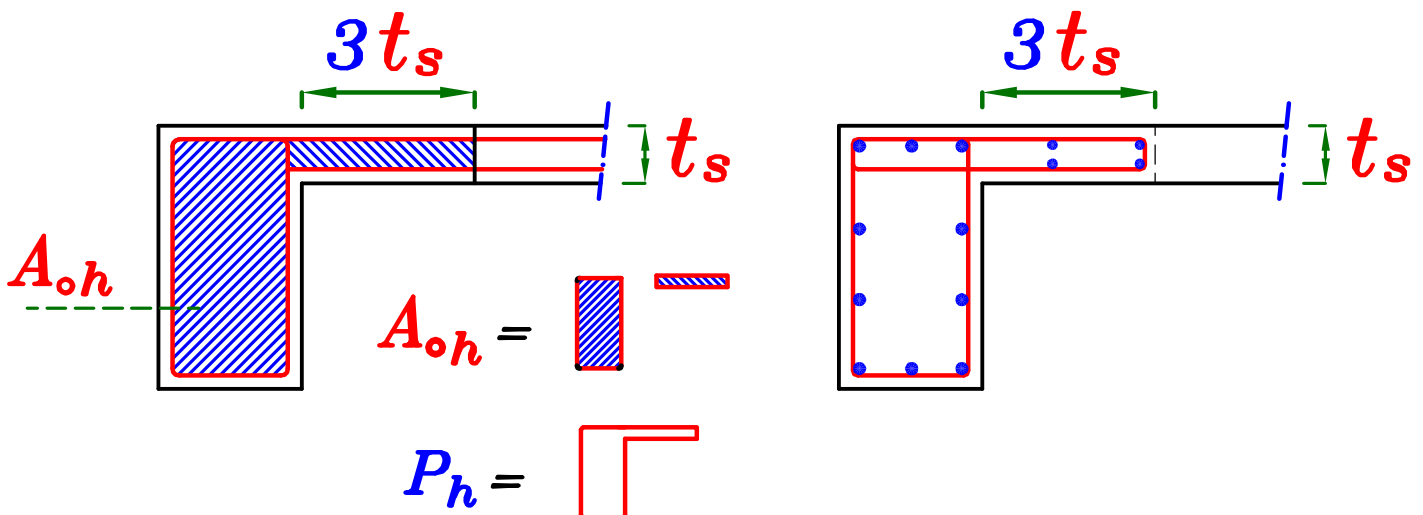
$$\therefore q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 * y_1^2)} \quad \text{For R-Sec. only}$$



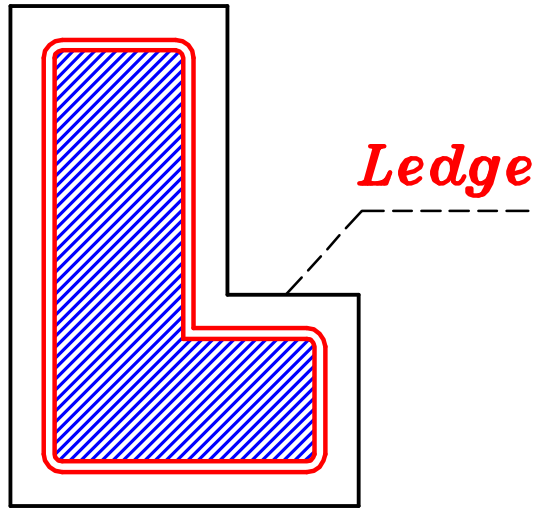
For L-Sec.

عند وجود بلاطه مع الكمره من الممكن أن نعتبر أن جزء من البلاطه يقاوم ال **Torsion** مع الكمره . و هذا الجزء يساوى تقريبا $3t_s$

بشرط ان يوضع فى هذا الجزء كانات لمقاومه ال **Torsion**



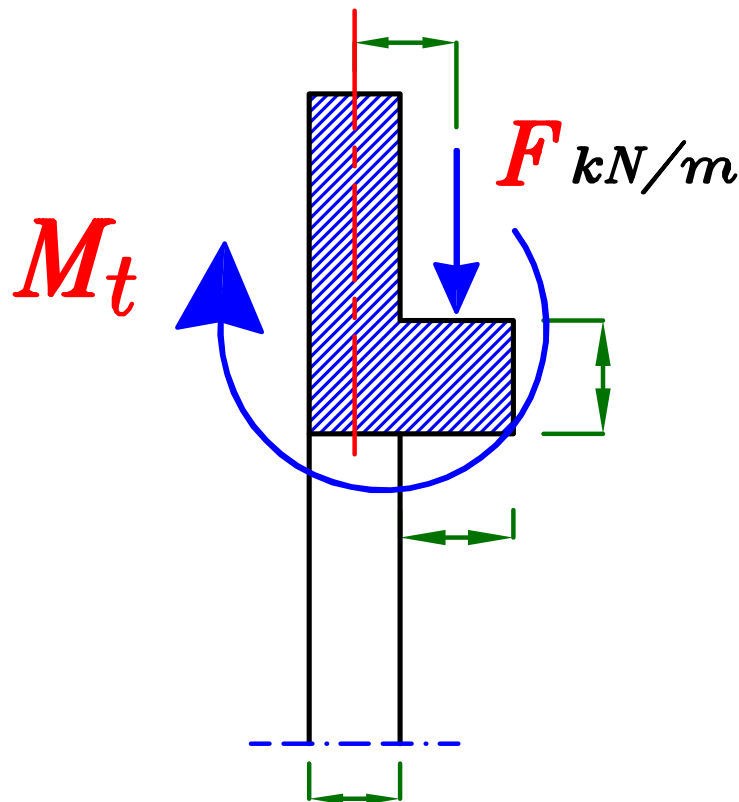
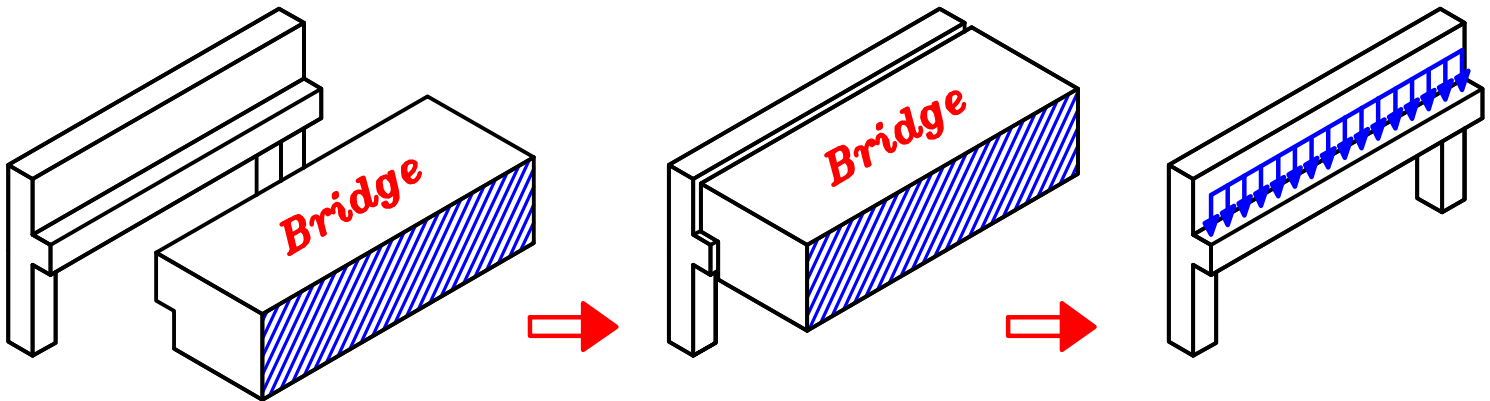
For Ledge Beam كمره ذات نتؤ



$$A_{oh} =$$

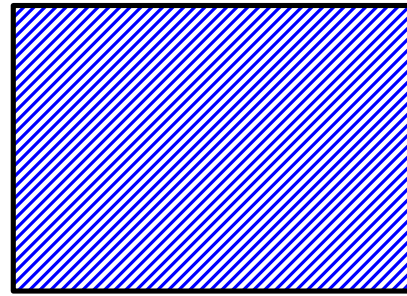
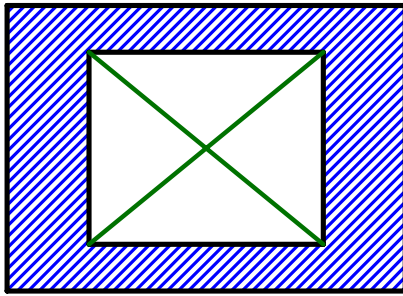
$$P_h =$$

عاده تستخدم **Ledge Beam** فى الكبارى



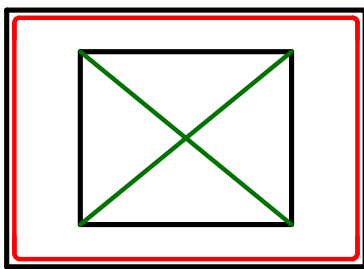
For Beams with Box Section. القطاعات الصندوقية

لان أكبر تأثير لـ **Torsion** يكون على الاطراف الخارجيه
و ال **Torsion stress** فى المنتصف يكون قليل
لذا تعتبر مقاومه القطاعات المفرغه من المنتصف لـ **Torsion** تقريبا تساوى
مقاومه القطاعات المصمته .

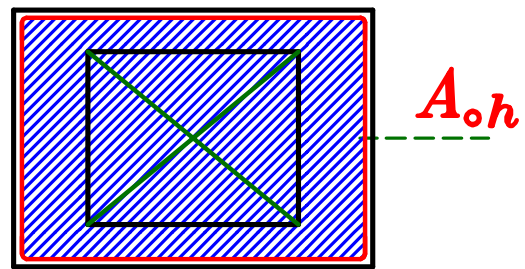


مقاومه ال **Torsion** \approx مقاومه ال **Torsion**

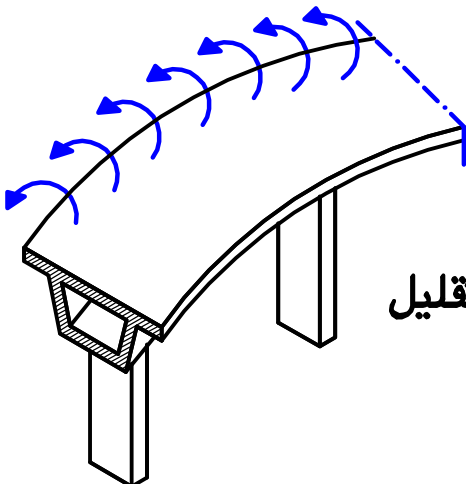
لذا فى القطاعات ال **Box Sections** عند حساب ال **Torsion**
نعتبره مثل القطاعات المصمته .



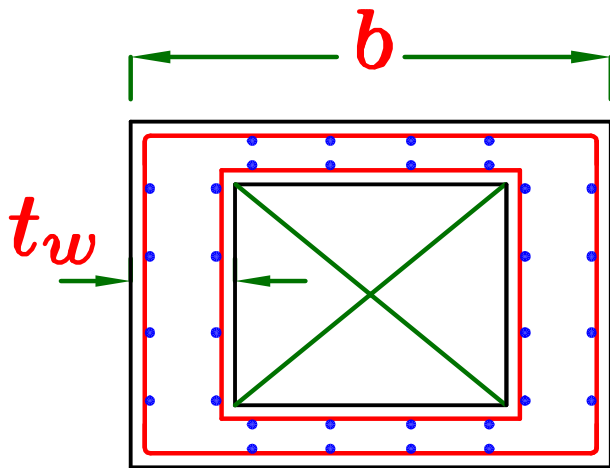
\approx



$$A_{oh} = \text{[hatched rectangle]} , P_h = \text{[empty rectangle]}$$



من اشهر تطبيقات ال **Box Sections**
يكون فى الكبارى التى بها منحنيات
لان مقاومتها لـ **Torsion** كبيره و فى نفس
الوقت وزنها خفيف فيقل ال **B.M.** مما يعمل على تقليل
التسليح و بالتالى توفير فى السعر .

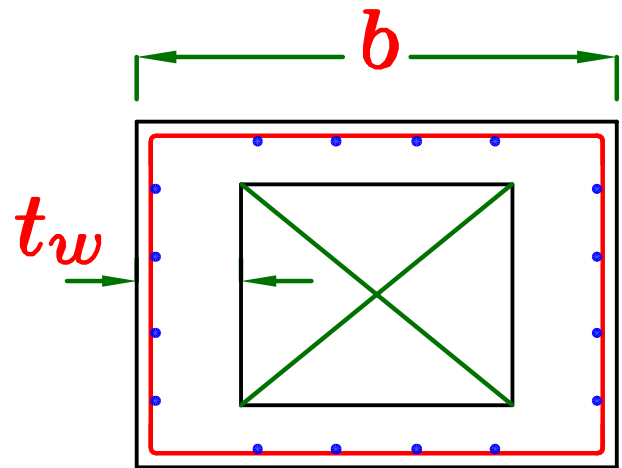


$$IF \quad t_w \leq \frac{b}{6}$$

نأخذ قيمه P_h هو طول الكانات
على المحيط الخارجى و الداخلى للقطاع

$$P_h =$$

يتم وضع اسياخ **Longitudinal Bars**
على المحيط الخارجى و الداخلى للقطاع



$$IF \quad t_w > \frac{b}{6}$$

نأخذ قيمه P_h هو طول الكانات
على المحيط الخارجى فقط

$$P_h =$$

يتم وضع اسياخ **Longitudinal Bars**
على المحيط الخارجى فقط

Design For Shear + Torsion.



Actual Stresses due to Shear Force. q_u

$$q_u = \frac{Q}{bd}$$

Actual Stresses due to Torsional Moment. q_{tu}

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

min. allowable stresses due to Shear q_{cu}

$$q_{cu} = (0.24) \sqrt{\frac{F_{cu}}{\delta_c}}$$

min. allowable stresses due to Torsion q_{tu}

$$q_{t_{min}} = (0.06) \sqrt{\frac{F_{cu}}{\delta_c}}$$

max. allowable shear stresses $q_{u_{max}}$

$$q_{u_{max}} = (0.70) \sqrt{\frac{F_{cu}}{\delta_c}}$$

IF $\sqrt{q_u^2 + q_{tu}^2} > q_{u_{max}} \rightarrow$ Increase Dimensions

For Box Sections only.

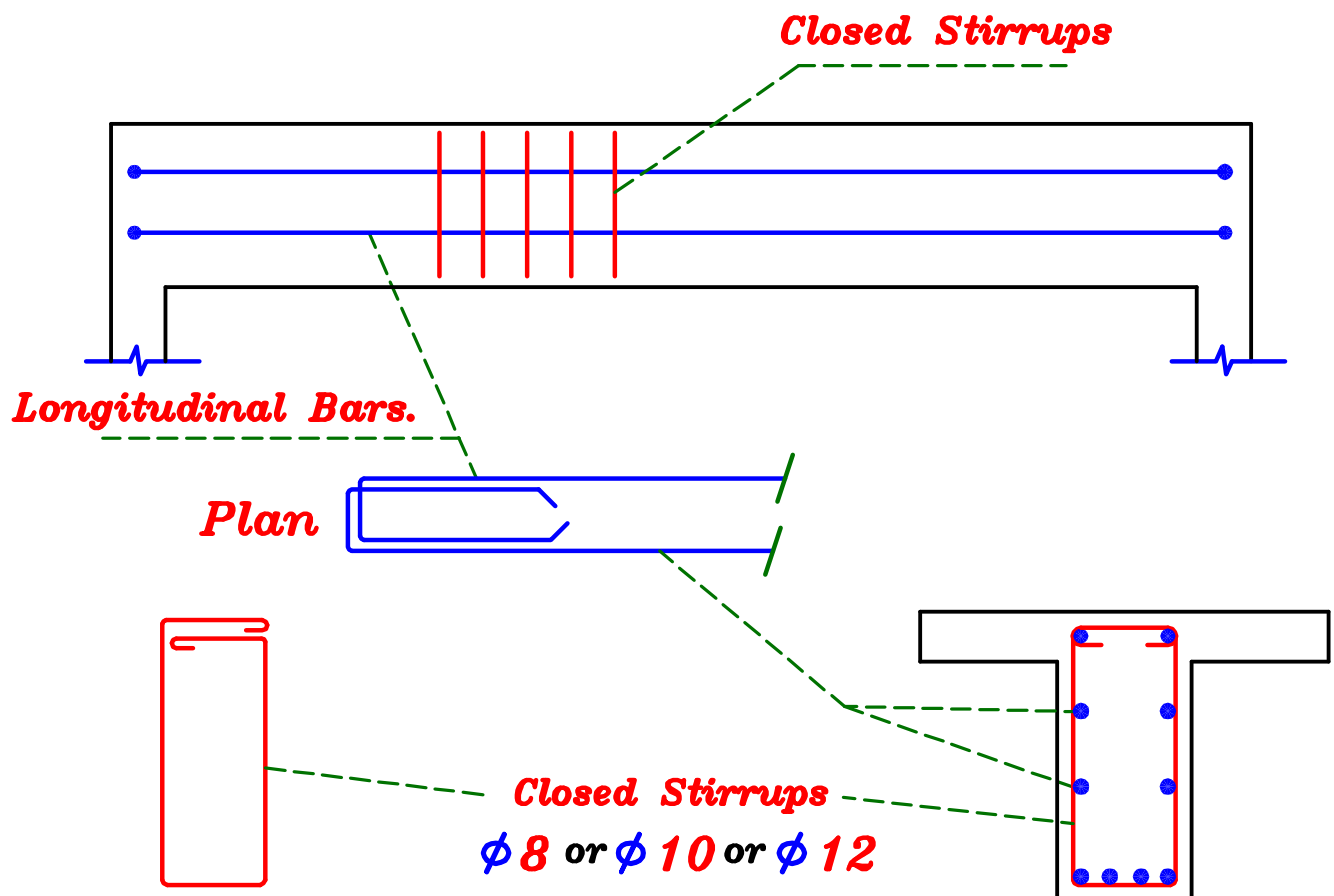
IF $q_u + q_{tu} > q_{u_{max}} \rightarrow$ Increase Dimensions

$$IF \sqrt{q_u^2 + q_{tu}^2} \leq q_{u \max}$$

	q_u	q_{tu}	$RFT.$
①	$q_u < q_{cu}$	$q_{tu} < q_{t \min}$	Use Stirrups $5 \phi 8 \setminus m$
②	$q_u > q_{cu}$	$q_{tu} < q_{t \min}$	Use RFT. to resist $(q_u - \frac{q_{cu}}{2})$
③	$q_u < q_{cu}$	$q_{tu} > q_{t \min}$	Use RFT. to resist (q_{tu})
④	$q_u > q_{cu}$	$q_{tu} > q_{t \min}$	Use RFT. to resist $(q_u - \frac{q_{cu}}{2}) + (q_{tu})$

How to Resist Torsion ??

- ① Closed Stirrups. ١ كانات مغلقة .
- ② Longitudinal Bars. ٢ أسياخ طوليه .



Case ③

$$q_u < q_{cu}, q_{tu} > q_{tmin}$$

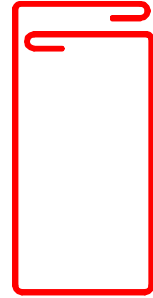
Use Shear RFT. to resist Shear Stresses $(q_{tu} - \frac{q_{tmin}}{2})$ applied From Torsional moment.



① Closed Stirrups.

$$A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\delta_s} \right)}$$

حفظ

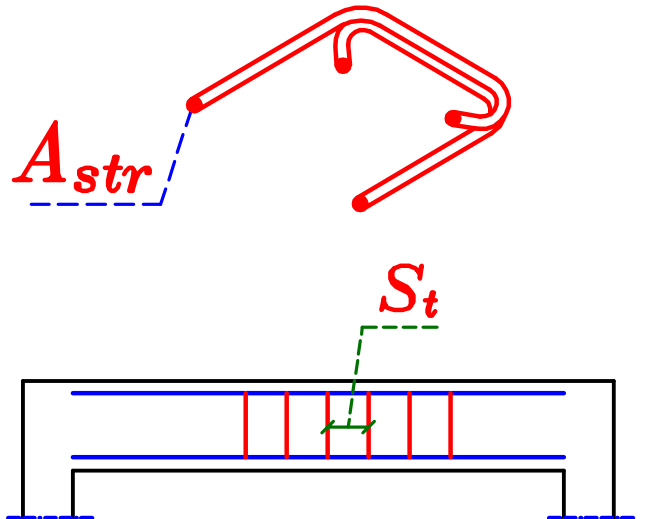


Closed Stirrup

Where:

* A_{str} مساحة مقطع سيخ الكانه

ϕ	A_{str}
$\phi 8$	50.3 mm ²
$\phi 10$	78.5 mm ²
$\phi 12$	113 mm ²



ملحوظه .

- ممكن استخدام كانات حتى $\phi 12$ فى ال **Torsion**

- كانات ال **Torsion** تكون الكانات الخارجيه فقط .

* S_t المسافه الطويله بين كانات ال **Torsion**

$$S_t = (100 \text{ mm} \rightarrow 200 \text{ mm})$$

In the equation choose $\phi \rightarrow A_{str} = \checkmark$

Then get $S_t = (100 \text{ mm} \rightarrow 200 \text{ mm})$

$$\left\{ \begin{array}{l} \text{IF } S_t \geq 200 \text{ mm} \xrightarrow{\text{use}} 5 \phi \checkmark \setminus m \\ \text{IF } S_t < 100 \text{ mm} \text{ Choose bigger } \phi \\ \text{IF } 100 \text{ mm} < S_t < 200 \text{ mm} \xrightarrow{\text{Get}} N_{\text{of stirrups}} \setminus m = \frac{1000}{S_t} \end{array} \right.$$

min. Stirrups.

ملاحظات .

$$\boxed{\frac{2 A_{str}}{b * S} < \frac{0.4}{F_y}} \rightarrow A_{str_{min}} = \frac{0.2 (b * S)}{F_y}$$

حيث F_y للكانات

$$S_{max.} = \frac{P_h}{8}$$

الاصغر

$$= 200 \text{ mm}$$

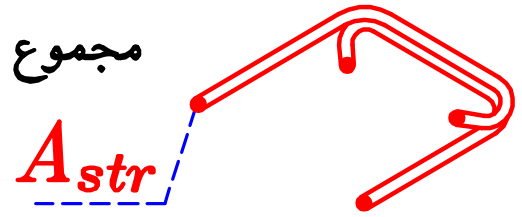
② Longitudinal Bars.

$$A_{sl} = \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y \text{ str.}}}{F_{y \text{ L.b.}}} \right)$$

Where:

* A_{sl} مجموع مساحة مقطع الأسيخ الطولية كلها.

* A_{str} مساحة مقطع سيخ الكانه.



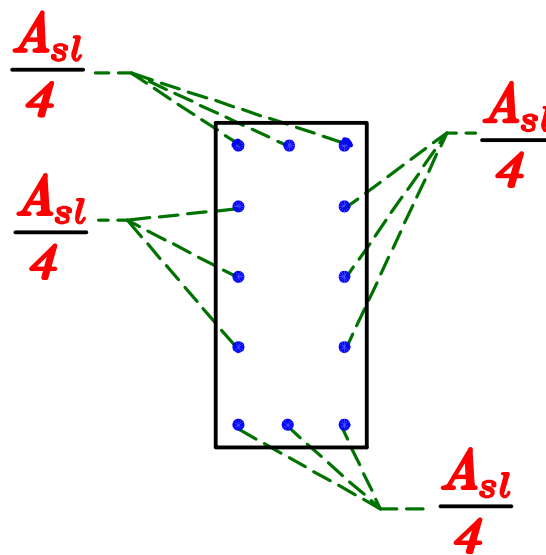
* $F_{y \text{ str.}} = F_y$ For stirrups. $\simeq 240 \text{ N/mm}^2$

* $F_{y \text{ L.b.}} = F_y$ For Longitudinal bars. $\simeq 360 \text{ N/mm}^2$

$$A_{sl \text{ min}} = \frac{0.4 \sqrt{\frac{F_{cu}}{\delta_c}}}{\frac{F_y}{\delta_s}} * A_c - \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y \text{ str.}}}{F_{y \text{ L.b.}}} \right)$$

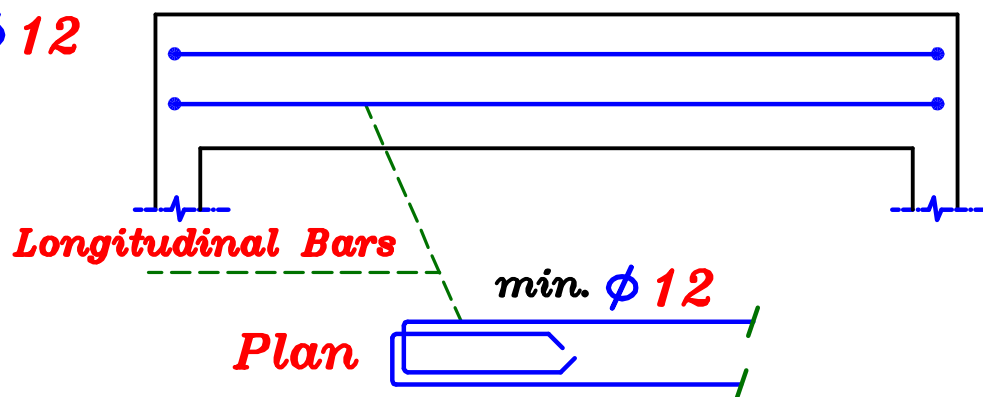
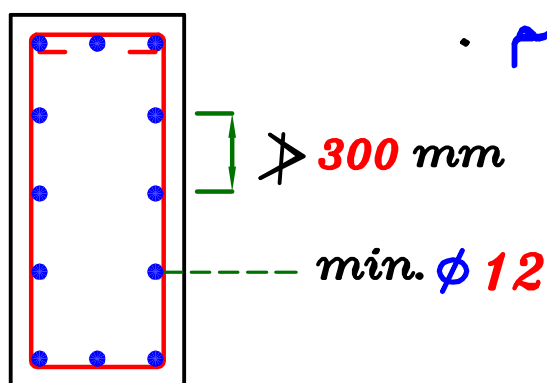
A_c هي المساحة الكلية للقطاع بما فيها الفتحات .

- توزيع الأسياخ على محيط القطاع بانتظام . $\frac{A_{sl}}{4}$



- المسافه بين الأسياخ لا تزيد عن ٣٠٠ مم .

- أقل قطر للسبخ $\phi 12$



إذا لرس تسليح ال **Longitudinal Bars**

يتم زياده مساحه $\frac{A_{sl}}{4}$ على مساحه التسليح الرئيسى للعزوم
ثم نحدد بعدها عدد الاسياخ الكليه و اقطارها .

يتم زياده مساحه $\frac{A_{sl}}{4}$ على مساحه ال **Stirrup Hangers**
ثم نحدد بعدها عدد الاسياخ الكليه و اقطارها .

يتم وضع اسياخ جانبيه بدل ال **Shrinkage Bars**

قيمتها $\phi 12$ 1 كل ٣٠٠ مم و توضع حتى لو كان $t < 700$ mm

Case ④

$$q_u > q_{cu} \& q_{tu} > q_{tmin}$$

Use Shear RFT. to resist Shear Stresses (q_{tu}) applied From Torsional moment.

+ Shear RFT. to resist Shear Stresses ($q_u - \frac{q_{cu}}{2}$) applied From Shear Force.

① Closed Stirrups.

① Torsion.



$$A_{str} = \frac{M_{tu} S_t}{(1.7) A_o h \left(\frac{F_y}{\delta_s} \right)}$$

$$A_{str} = \checkmark * S \quad \text{--- ①}$$

كانات خارجيه فقط

A_{str} هي مساحه سيخ الكانه الخارجيه التي نحتاجها لمقاومه ال **Torsion** فقط.

② Shear.

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y / \delta_s)}{b S_s}$$

$$A_s = \checkmark * \frac{S}{n} \quad \text{--- ②}$$

كانات خارجيه و ممكن كانات داخله

A_s هي مساحه مقطع سيخ واحد من الكانه الخارجيه أو الداخليه التي نحتاجها لمقاومه ال **Shear** فقط.

— نبدأ أولاً بفرض أن عدد فروع الكانه يساوى فرعين $n = 2$ و عدد الكانات في المتر = من (0 ← 10) أسياخ/م ثم نحسب $S = \frac{1000}{\text{No of st./m}}$ ثم نحسب A_{str} ، A_s

و تكون مساحه السيخ من الكانات الخارجيه $A_{s_{outer}} = A_{str} + A_s$

— فاذا كانت $A_{s_{outer}} \leq 113 \text{ mm}^2$ أى أن $\phi_{outer} \leq \phi 12$

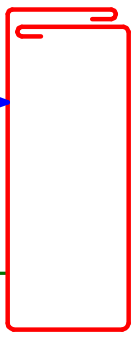
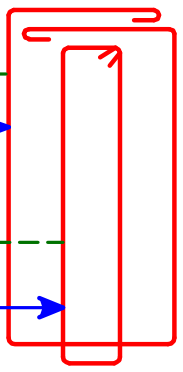
فنختار $\phi_{outer} = \phi 8 \text{ or } \phi 10 \text{ or } \phi 12$ و لا توجد كانات داخله

— أما اذا كانت $A_{s_{outer}} > 113 \text{ mm}^2$ فنختار عدد كانات أكثر في المتر أو نأخذ $n = 4$

ثم تحديد قيمه A_s و تحديد قيمه $A_{s_{outer}} = A_{str} + A_s$

و تحديد قيمه $A_{s_{inner}} = A_s$

ترتيب اختيار الفروض n, S

Assumption No.	n	No. of stirrups\m	$S_s = S_t$ (mm)	
1	2	5.0	$\frac{1000}{5.0}$	<div style="text-align: center;"> <p>الكانات الخارجيه تقاوم</p> <p>$\xrightarrow{\text{Shear + Torsion}}$</p> <p>$\phi_{outer}$</p>  </div>
2	2	6.0	$\frac{1000}{6.0}$	
3	2	7.0	$\frac{1000}{7.0}$	
4	2	8.0	$\frac{1000}{8.0}$	
5	2	9.0	$\frac{1000}{9.0}$	
6	2	10	$\frac{1000}{10}$	
7	4	5.0	$\frac{1000}{5.0}$	<div style="text-align: center;"> <p>ϕ_{outer}</p> <p>الكانات الخارجيه تقاوم</p> <p>$\xrightarrow{\text{Shear + Torsion}}$</p> <p>$\phi_{Inner}$</p> <p>الكانات الداخليه تقاوم</p> <p>$\xrightarrow{\text{فقط Shear}}$</p>  </div>
8	4	6.0	$\frac{1000}{6.0}$	
9	4	7.0	$\frac{1000}{7.0}$	
10	4	8.0	$\frac{1000}{8.0}$	
11	4	9.0	$\frac{1000}{9.0}$	
12	4	10	$\frac{1000}{10}$	

min. Stirrups.

$$\frac{2 A_{str} + n A_s}{b * S} \leq \frac{0.4}{F_y}$$

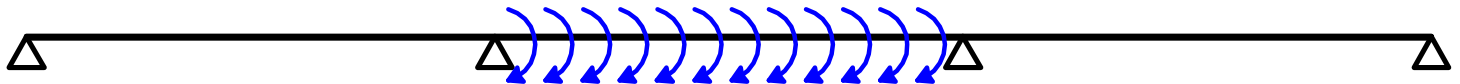
② Longitudinal Bars. Torsion only

$$A_{sl} = \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y \text{ str.}}}{F_{y \text{ L.b.}}} \right)$$

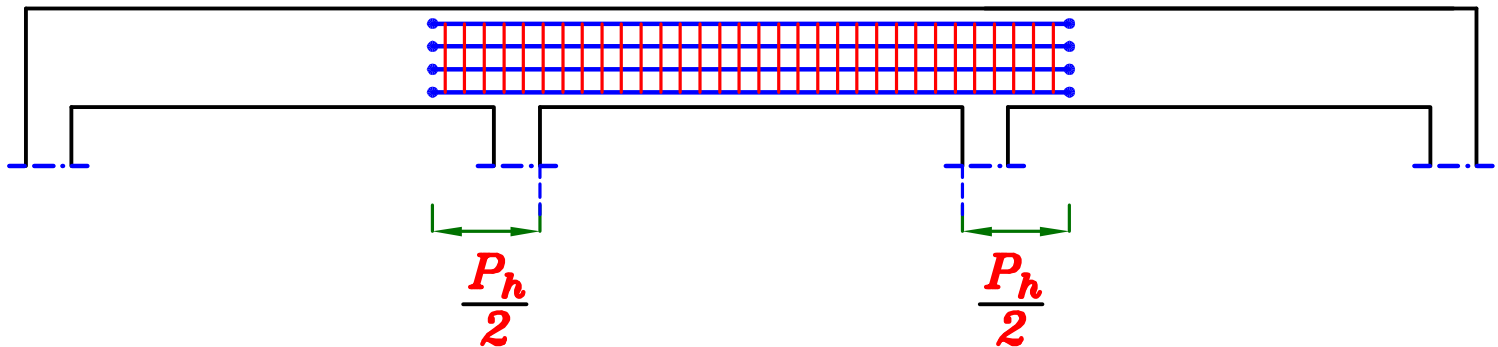
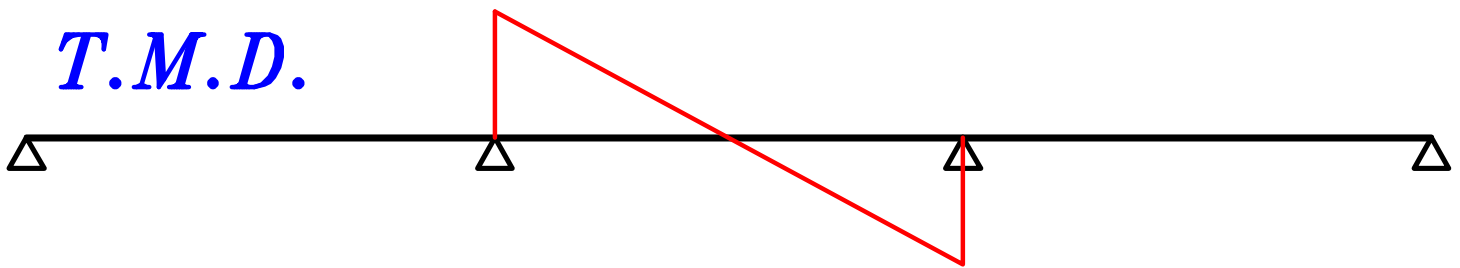
ملحوظه .

يجب ان تمتد كانات ال **Torsion** و اسياخ ال **Longitudinal Bars** مسافه لا تقل عن $\frac{P_h}{2}$ بعد اخر قطاع عليه **Torsional moment**

m_t



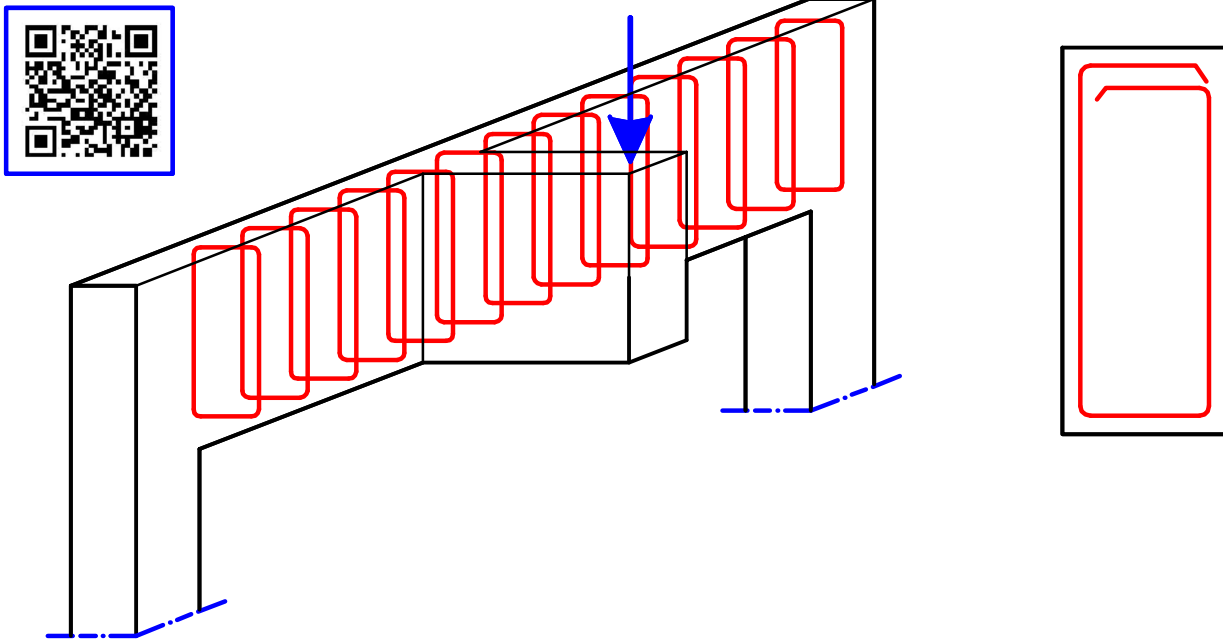
T.M.D.



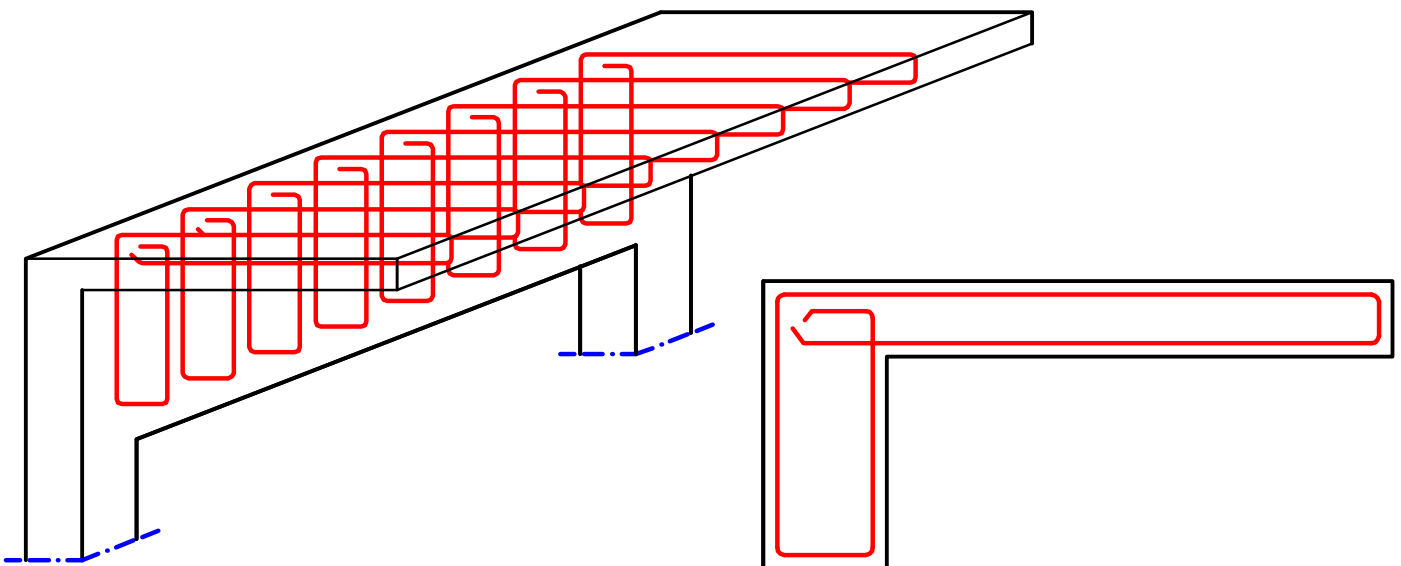
Check or Design of Torsion.

توجد حالتان لكانات ال *torsion* :

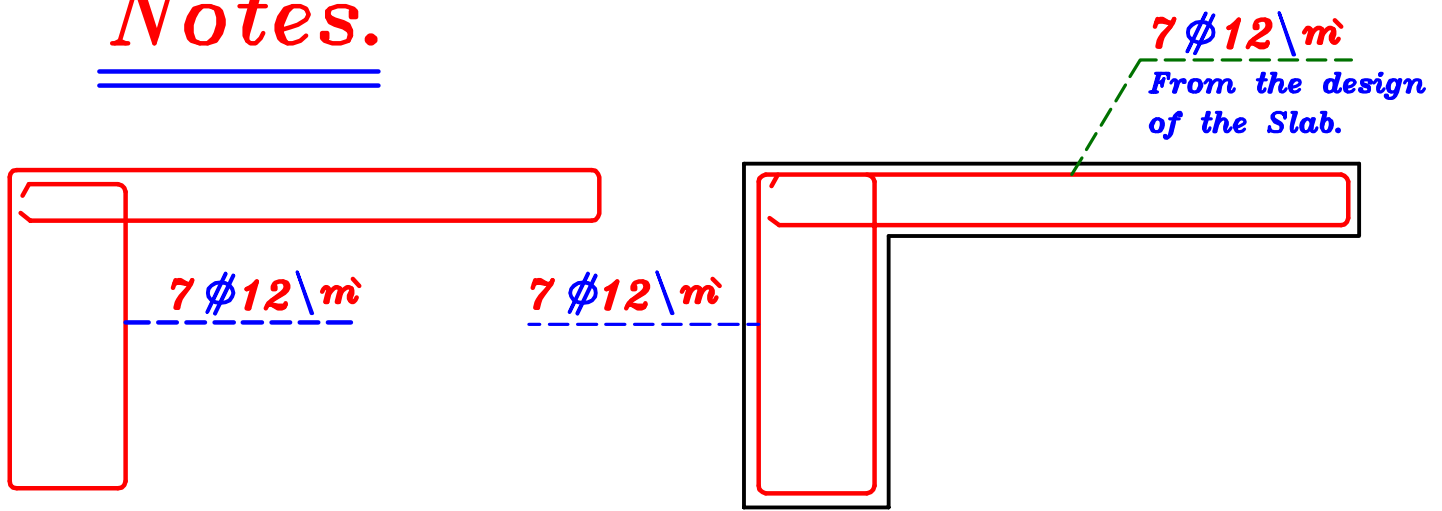
١- اذا لم يوجد تسليح للبلاطه يعمل كانات الكمره
فى هذه الحاله سيتم تصميم كانات لمقاومه ال *torsion*



٢- اذا كان تسليح البلاطه يعمل ككانات للكمرة .
فى هذه الحاله تعتبر كانات الكمره معطاه (و هى تسليح البلاطه) .
لذا سنعمل *Check* اذا كانت كانات الكمره كافيه ام سنحتاج لزيادتها .



Notes.



إذا كان تسليح البلاطة $7 \phi 12 \backslash m$ ← الكانات $7 \phi 12 \backslash m$ سنأخذ عدد الكانات الخارجيه $V =$ كانات في المتر و فرعان للكانه و $F_y = 360 \text{ N/mm}^2$ لا نأخذ كانات داخلية .

ثم نعمل **Check** إذا كانت $A_{s_{outer}} = \phi 12 = 113 \text{ mm}^2$ تكفى لتحمل ال **Shear Stress** أم أننا سنحتاج إلى إستخدام كانات داخلية .

For Torsion Eqn. $A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\delta_s} \right)} \rightarrow A_{str} = \checkmark$

$S_t = \frac{1000}{7.0}$, $F_y = 360 \text{ N/mm}^2$

For Shear Eqn. $q_u - \frac{q_{ou}}{2} = \frac{n A_s (F_y \backslash \delta_s)}{b S_s} \rightarrow A_s = \checkmark$

$n = 2$, $S_s = \frac{1000}{7.0}$, $F_y = 360 \text{ N/mm}^2$

After we get A_s, A_{str} chek $A_{s_{outer}} = A_{str} + A_s \leq 113 \text{ mm}^2$

* IF $A_{s_{outer}} \leq 113 \text{ mm}^2 \therefore \text{O.K.}$



تسليح البلاطة هو نفس تسليح الكانه الخارجيه

* IF $A_{s_{outer}} > 113 \text{ mm}^2$

\therefore use another RFT. system

or use choose another ϕ_{Inner}



نختار كانات داخلية عددها V في المتر

Example.

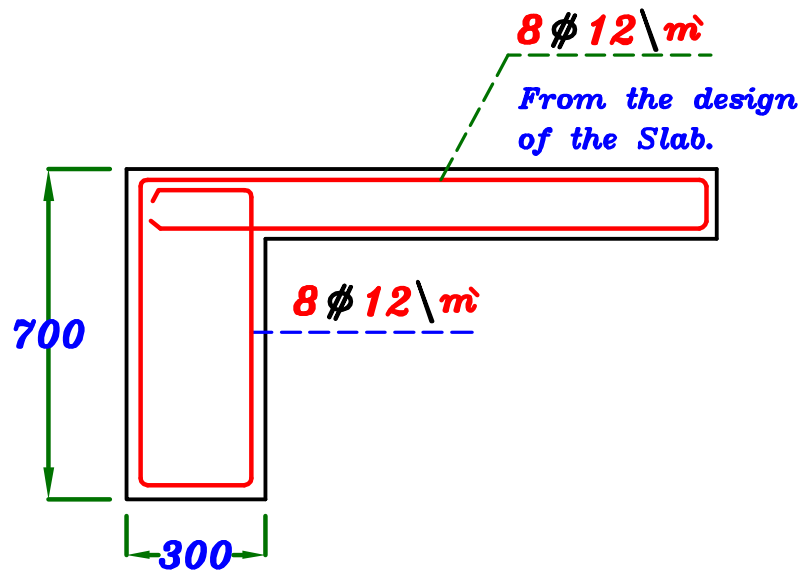
$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

$$M_{tu} = 55.8 \text{ kN.m}$$

$$q_u = 1.06 \text{ N/mm}^2$$

$$q_t = 2.38 \text{ N/mm}^2$$



RFT. of the cantilever Slab 8 #12 \ m

Req.

Check the Stirrups For Shear & Torsion.

Solution.

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2, \quad q_{t \min} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245 \text{ N/mm}^2$$

$$q_{u \max} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85 \text{ N/mm}^2$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{1.06^2 + 2.38^2} = 2.60 < q_{u \max} \therefore \text{O.k.}$$

$q_u > q_{cu}, \quad q_{tu} > q_{t \min}$	resist $(q_u - \frac{q_{cu}}{2}) + (q_{tu})$
-------------------------------------------	-------------------------------------------------

@Torsion.

$$x_1 = 220 \text{ mm}, \quad y_1 = 620 \text{ mm}, \quad A_{oh} = 220 * 620 = 136400 \text{ mm}^2$$

$$A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\delta_s} \right)} \therefore A_{str} = \frac{(55.8 * 10^6) S_t}{(1.7) (136400) (360 / 1.15)}$$

$$\therefore A_{str} = 0.768 S_t$$

⑥ Shear.

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \setminus \gamma_s)}{b S_s} \therefore 1.06 - \frac{0.98}{2} = \frac{n A_s (360/1.15)}{(300) S_s}$$

$$\therefore A_s = 0.546 \frac{S_s}{n}$$

$$n = 2, S_t = S_s = \frac{1000}{8.0} = 125 \text{ mm}$$

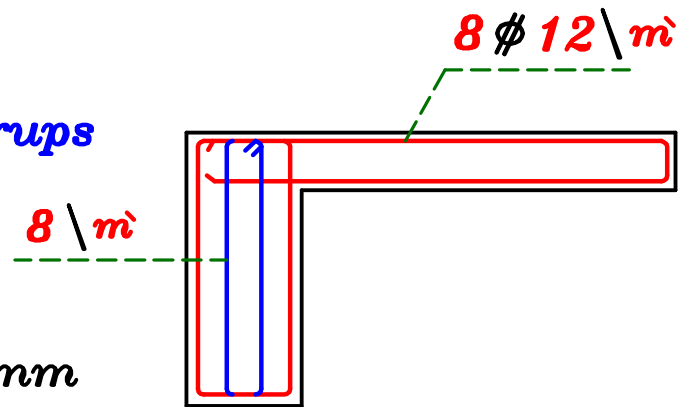
$$\therefore A_{str} = 0.768 S_t = 0.768 * 125 = 96.0 \text{ mm}^2$$

$$A_s = 0.546 \frac{S_s}{n} = 0.546 * \frac{125}{2.0} = 34.12 \text{ mm}^2$$

$$A_{s_{outer}} = A_{str} + A_s = 96.0 + 34.12 = 130.12 > 113$$

\therefore We have to use Inner Stirrups

$$n = 4, S_t = S_s = \frac{1000}{8.0} = 125 \text{ mm}$$

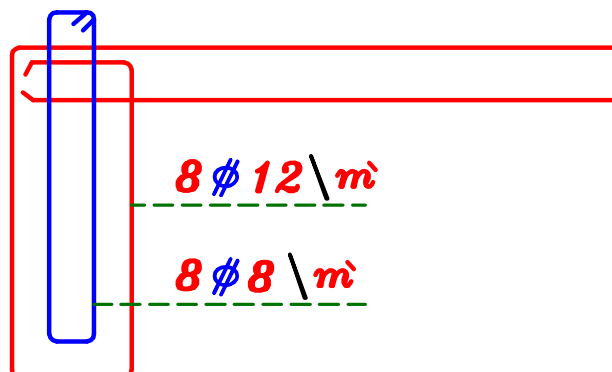


$$\therefore A_{str} = 0.768 S_t = 0.768 * 125 = 96.0 \text{ mm}^2$$

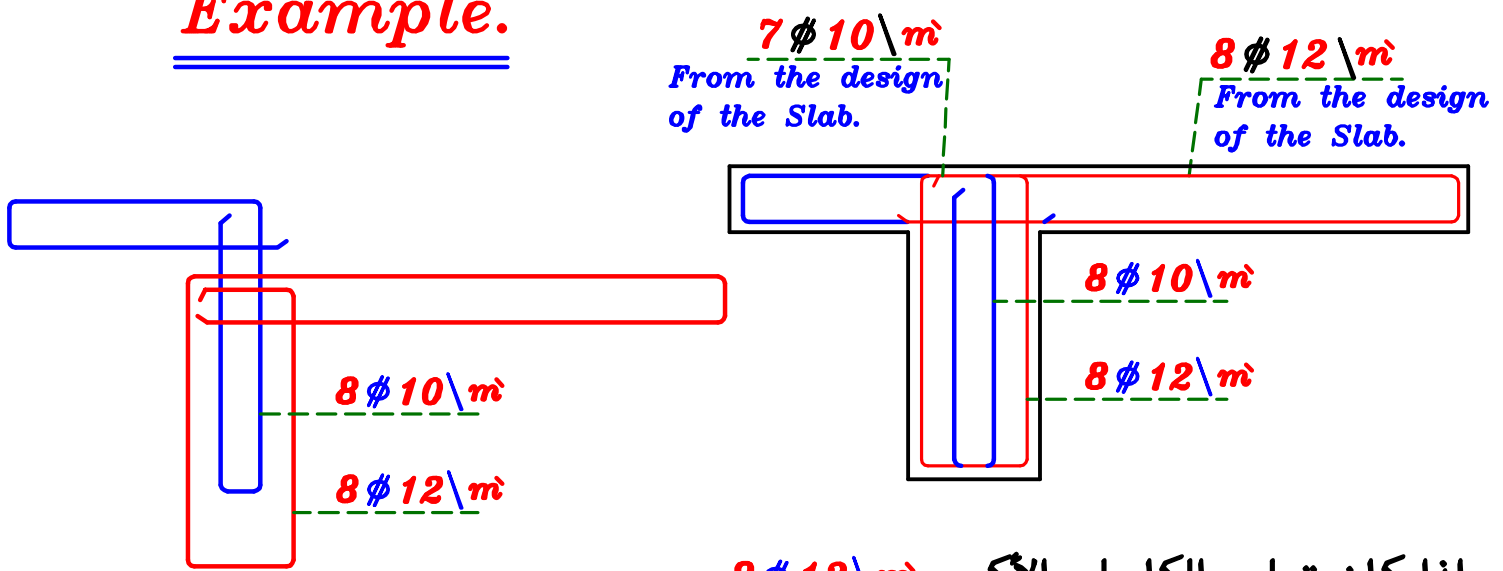
$$A_s = 0.546 \frac{S_s}{n} = 0.546 * \frac{125}{4.0} = 17.0 \text{ mm}^2$$

$$A_{s_{outer}} = A_{str} + A_s = 96.0 + 17.0 = 113 \leq 113 \therefore \text{O.K.}$$

$$A_{s_{inner}} = A_s = 17.0 \text{ mm}^2 = \phi 8$$



Example.



إذا كان تسليح الكابولي الأكبر 8 #12 \ m

سنأخذ الكانات الخارجيه 8 #12 \ m $F_y = 360 \text{ N/mm}^2$

إذا كان تسليح الكابولي الأصغر 7 #10 \ m سوف نأخذه 8 #10 \ m و نأخذ

الكانات الداخليه 8 #10 \ m لكى يكون عدد الكانات الداخليه و الخارجيه متساوى

ثم نعمل **Check** إذا كانت $A_{s_{inner}} \leq \phi 10 = 78.5 \text{ mm}^2$ و $A_{s_{outer}} \leq \phi 12 = 113 \text{ mm}^2$

تكفى لتحمل ال **Shear Stress**

For Torsion Eqn.

$$F_y = 360 \text{ N/mm}^2, S_t = \frac{1000}{8.0}$$

$$A_{str} = \frac{M_{tu} S_t}{(1.7) A_o h \left(\frac{F_y}{\delta_s} \right)} \rightarrow A_{str} = \checkmark$$

For Shear Eqn.

$$n = 4, F_y = 360 \text{ N/mm}^2, S_s = \frac{1000}{8.0}$$

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \delta_s)}{b S_s} \rightarrow A_s = \checkmark$$

After we get A_s, A_{str}

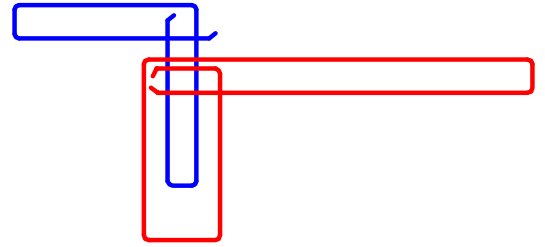
$$\text{Get } A_{s_{outer}} = A_{str} + A_s \leq 113 \text{ mm}^2$$

$$A_{s_{inner}} = A_s \leq 78.5 \text{ mm}^2$$

Check.

$$* \text{ IF } A_{s_{outer}} \leq 113 \text{ mm}^2$$

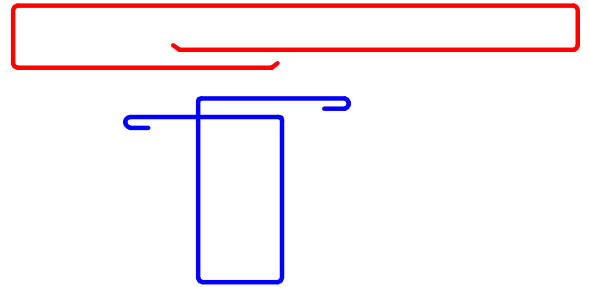
$$A_{s_{inner}} \leq 78.5 \text{ mm}^2$$



\therefore **O.K.** تسليح البلاطه هو نفس تسليح الكانه الخارجيه

$$* \text{ IF } A_{s_{outer}} > 113 \text{ mm}^2$$

$$\text{or } A_{s_{inner}} > 78.5 \text{ mm}^2$$

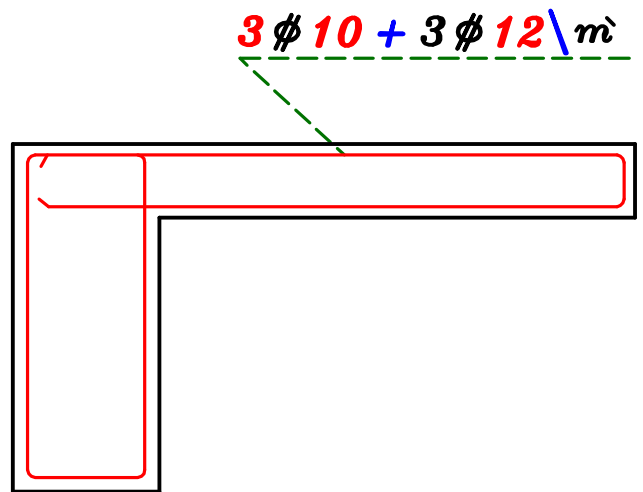


Increase Dimensions

or use another RFT. system

ليس شرط أن يكون تسليح البلاطه
هو نفس تسليح الكانه الخارجيه

Example.

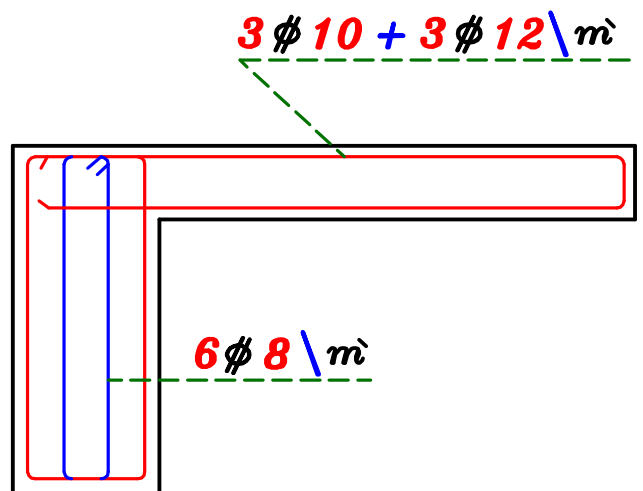
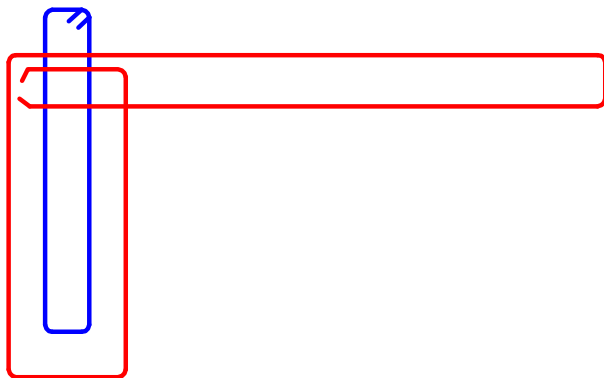


For Shear Eqn. $n = 2$, $S_s = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

For Torsion Eqn. $S_t = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

, Check $A_{s_{outer}} = A_s + A_{str} \leq \left(\frac{78.5 + 113}{2} \right)$

Example.



For Shear Eqn. $n = 4$, $S_s = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

For Torsion Eqn. $S_t = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

Check $A_{s_{inner}} = A_s \leq 50.3$

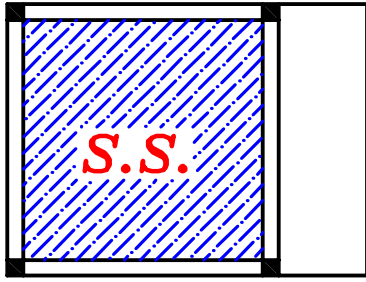
, Check $A_{s_{outer}} = A_s + A_{str} \leq \left(\frac{78.5 + 113}{2} \right)$

Special Cases.

إذا وجد كابولي بجوار بلاطة الحمام

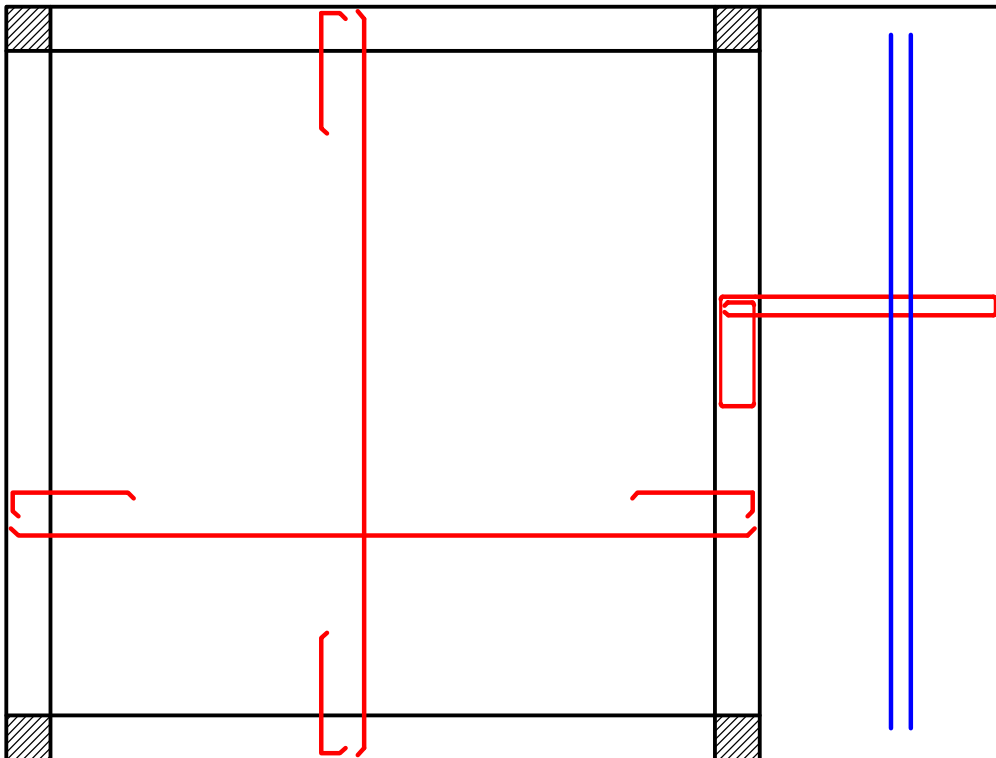
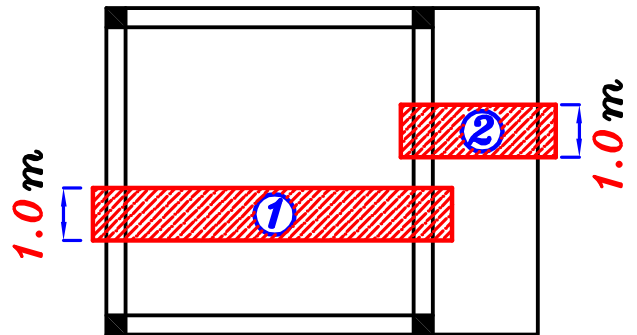
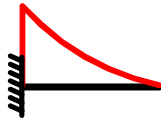
يوجد طريقتين للتسليح :

① نأخذ كل شريحة بمفردها



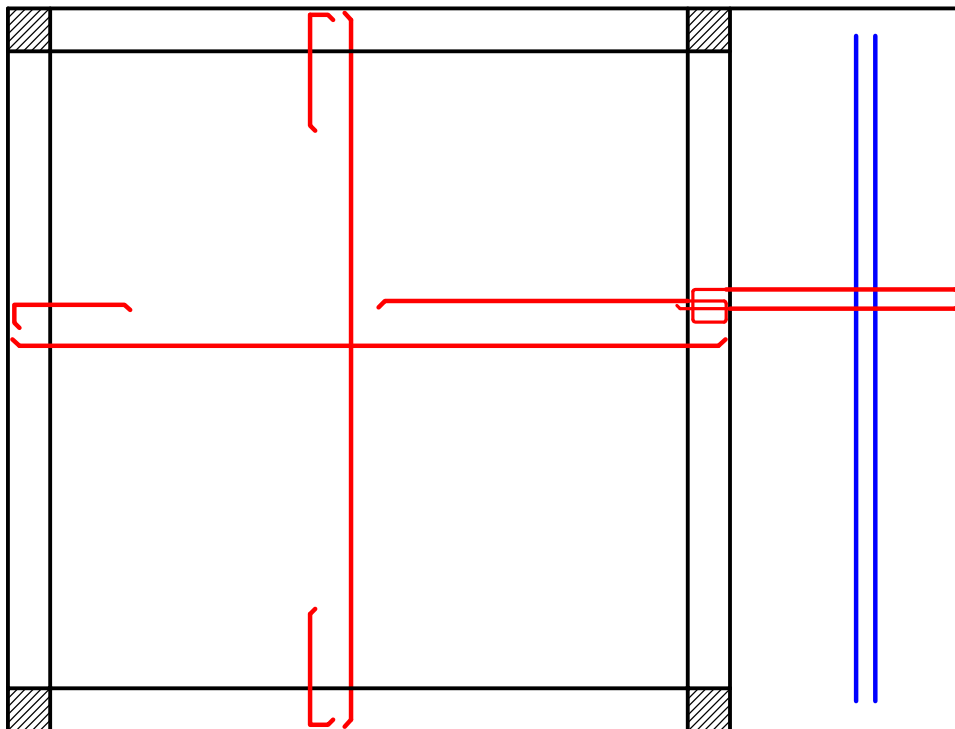
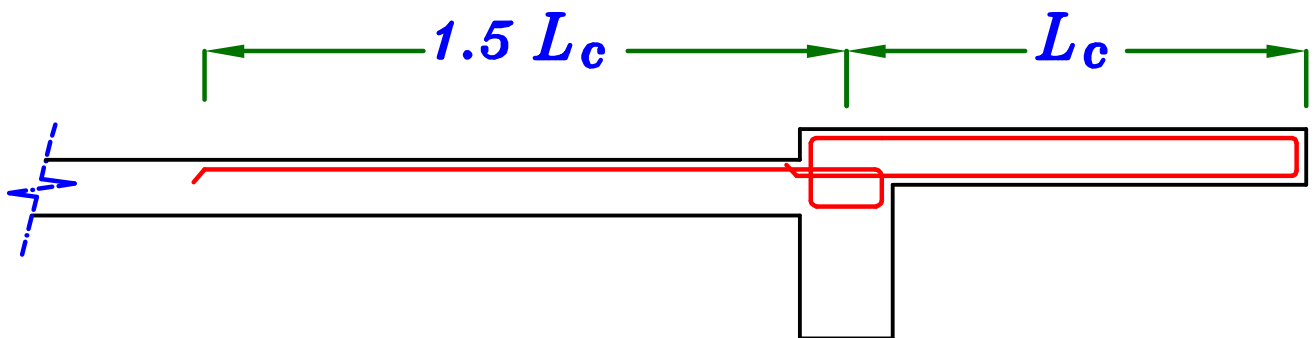
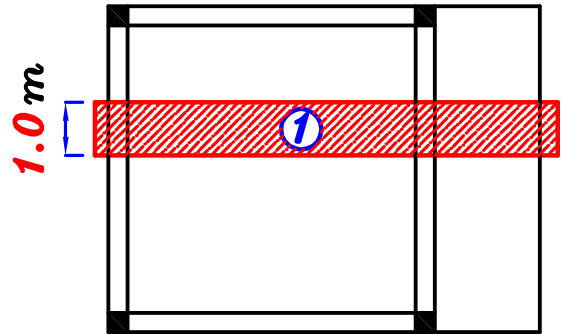
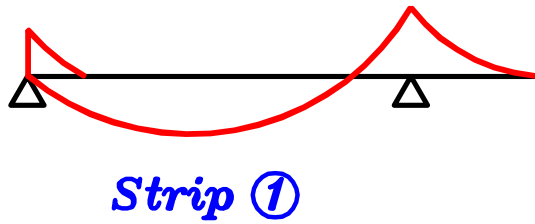
Strip ①

Strip ②

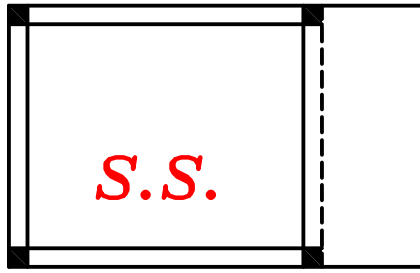


② نأخذ شريحه على البلاطه و الكابولى معا

فتكون محمولة على أكثر من *one support* فلا يتكون *Torsion* على الكمره



Special Case.



إذا وجد كابولي سفلي كما بالشكل

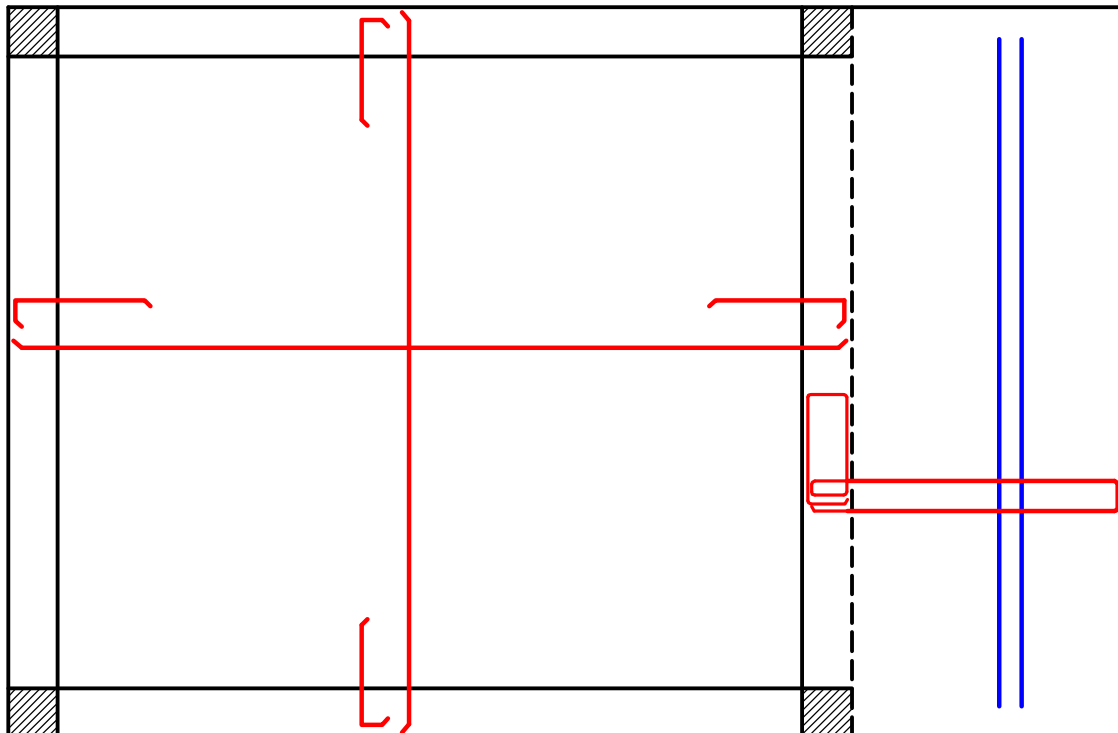
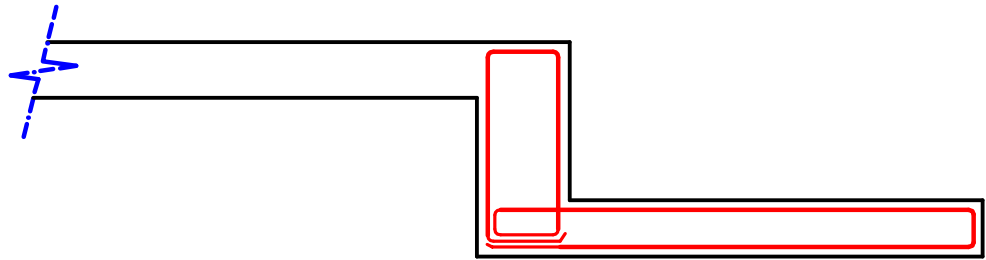
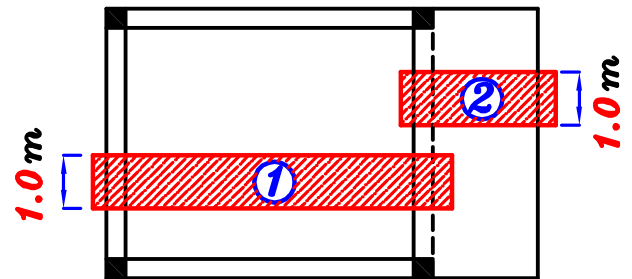
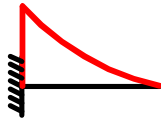
يوجد طريقتين للتسليح :

① نأخذ كل شريحة بمفردها



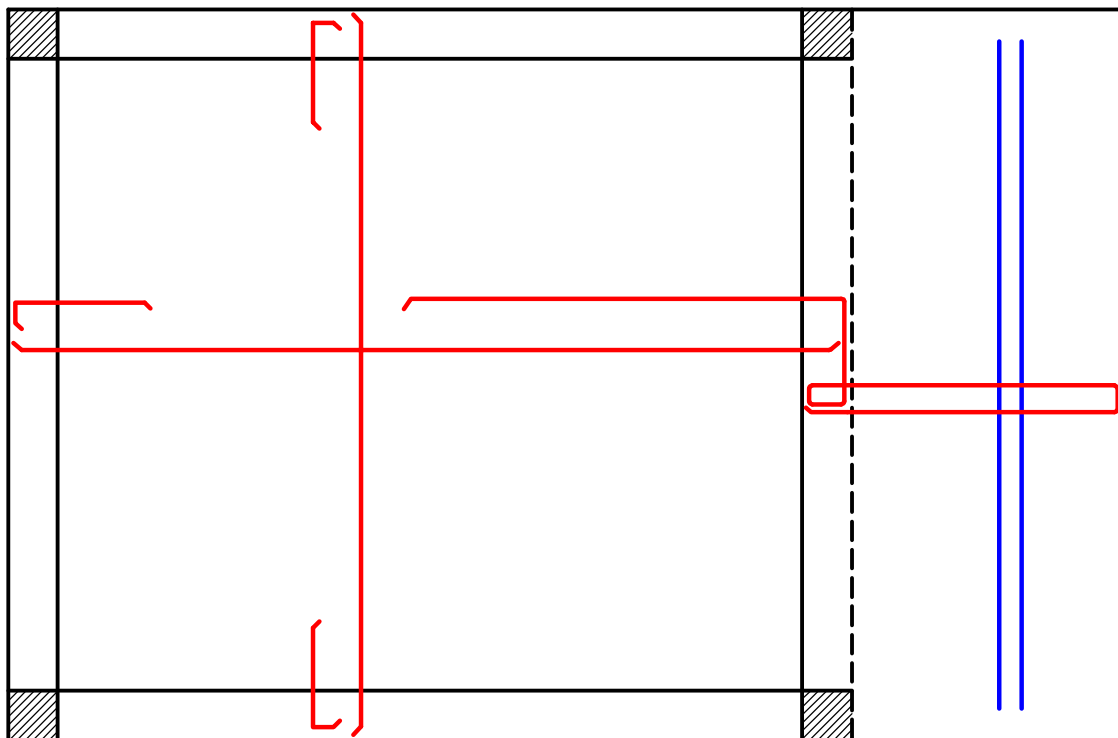
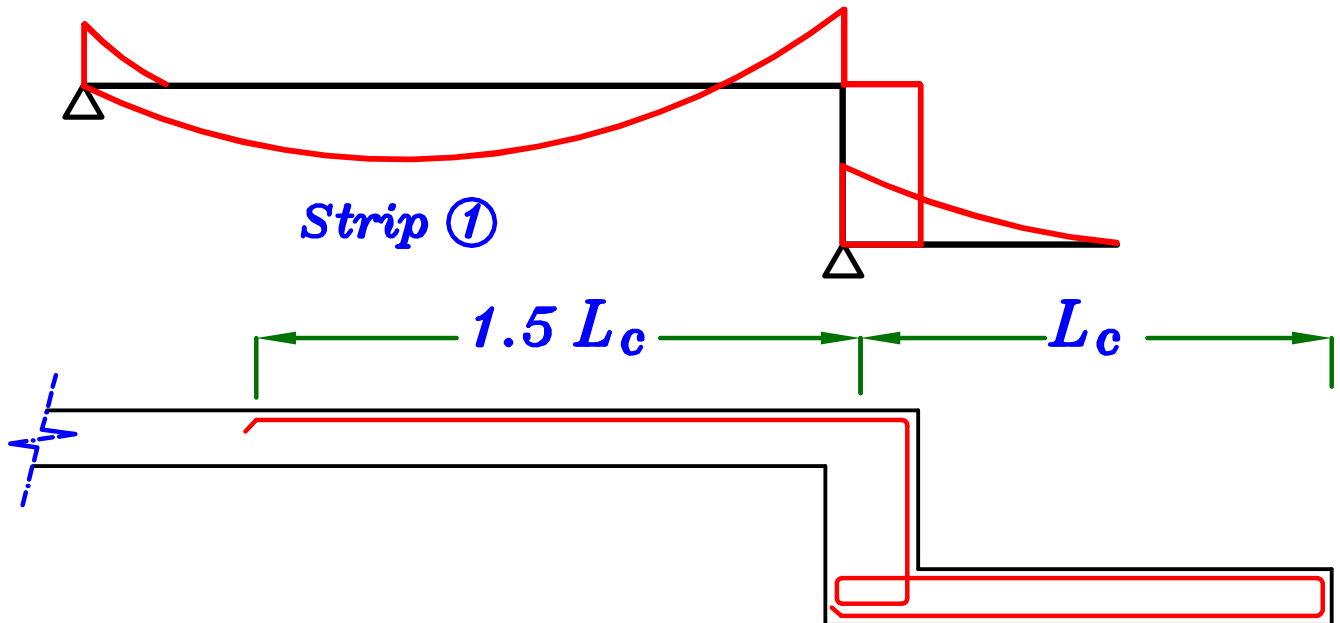
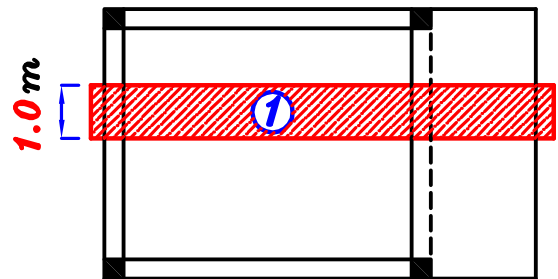
Strip ①

Strip ②



② نأخذ شريحه على البلاطه و الكابولى معا

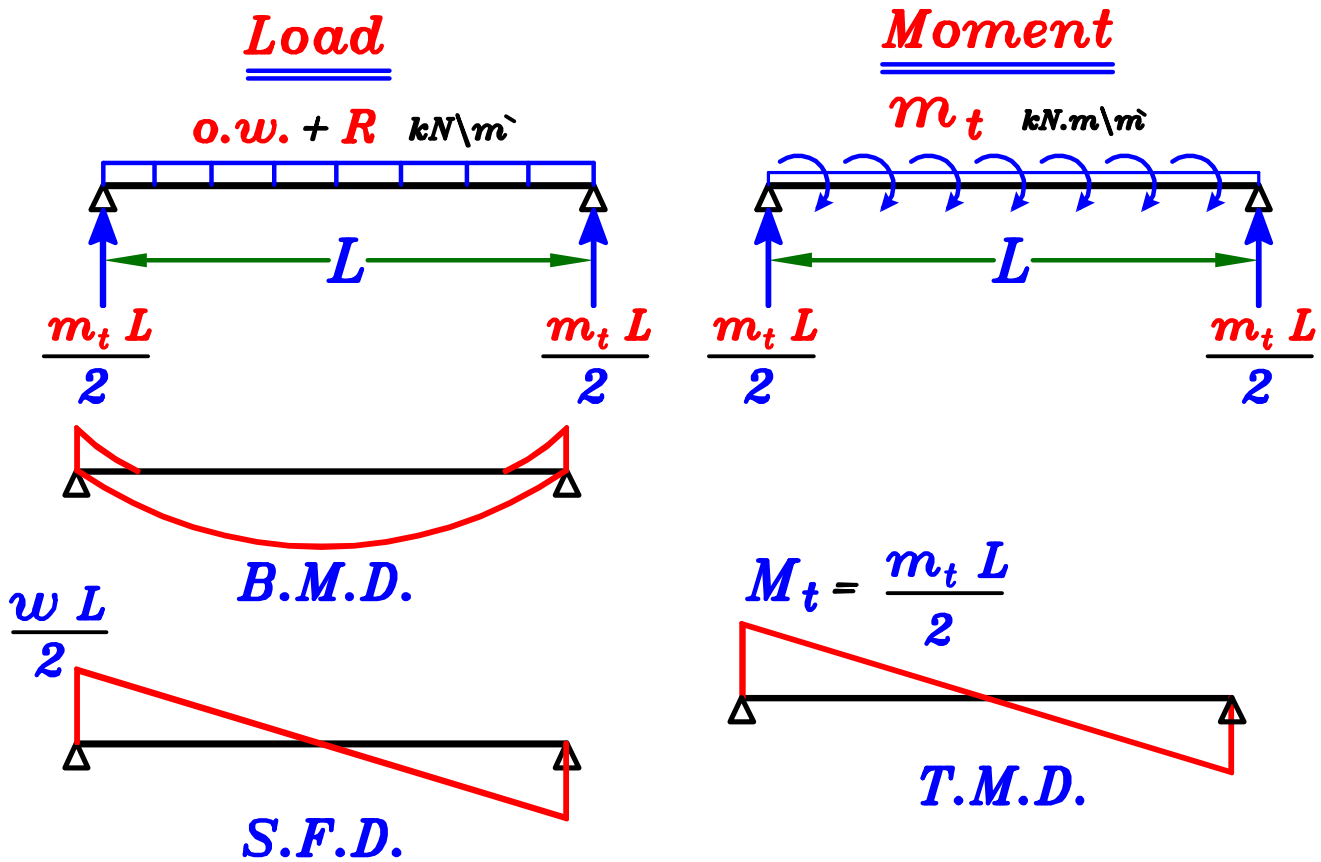
فتكون محموله على أكثر من *one support* فلا يتكون *Torsion* على الكمره



Steps of design a Beam Subjected to Torsional Moment.



① Draw B.M.D. , S.F.D. & T.M.D.



② Estimate the Dimensions Of the Beam (b, t)

① $b = 300 \text{ mm}$ OR 350 mm OR 400 mm

② Get t

$$t_{tor.} \approx \frac{3 M_t}{1.6 b^2}$$

$$t_{ben.} = 3.50 \sqrt{\frac{M}{F_{cu} b}}$$

Take $t =$ The bigger value of $t_{tor.}, t_{ben.}$

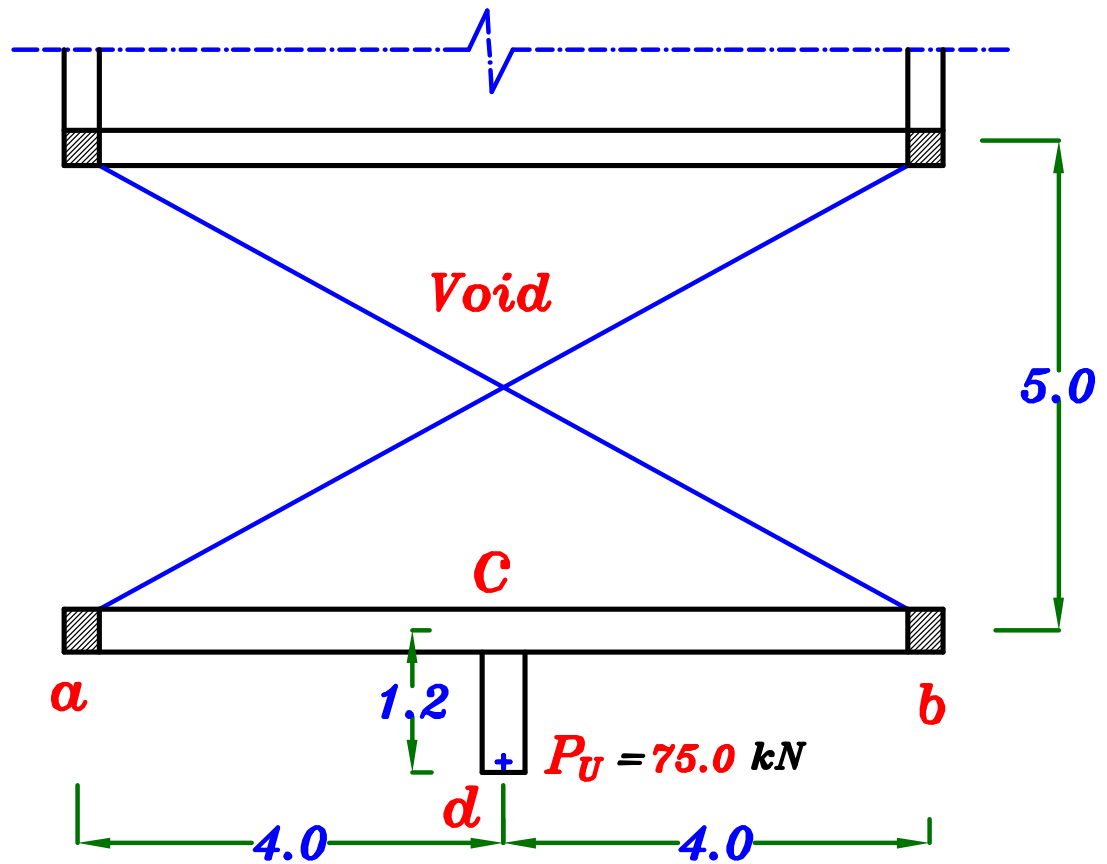
③ Design the beam according to B.M.
and get the main steel (A_s)

④ Design the beam according to Shear & Torsion.
and get Stirrups & Longitudinal Bars.

Examples on Torsion.



Example.



Data.

$$F_{cu} = 25 \text{ N/mm}^2 \quad F_y = 360 \text{ N/mm}^2$$

Req.

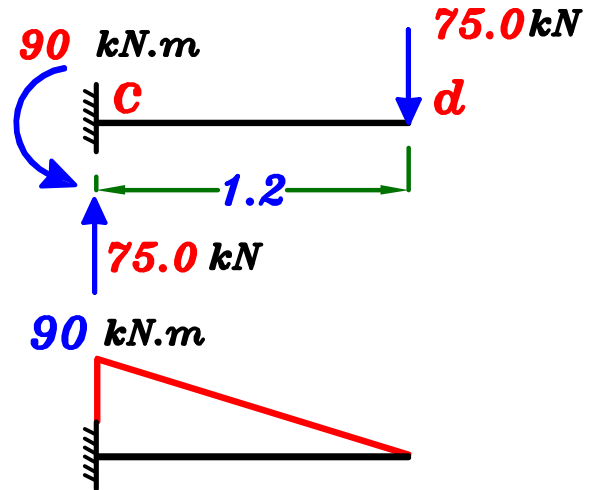
- ① Design the beam (a b) and Cantilever (c d) (Neglecting the O.W.) and draw The details of RFT.
- ② Get Loads on Column (b) at this case. and Draw the Internal Forces Diagrams For that Column.
- ③ Draw a necessary member to make beam (a b) Free of Torsion. and Draw the Internal Forces Diagrams For the Beam at this case.

Solution.

①

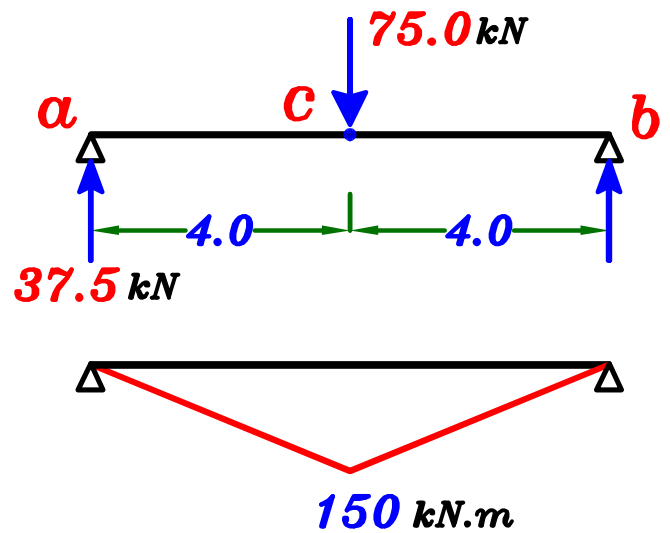
Loads on
The Cantilever (c d)

B.M.D. (c d)

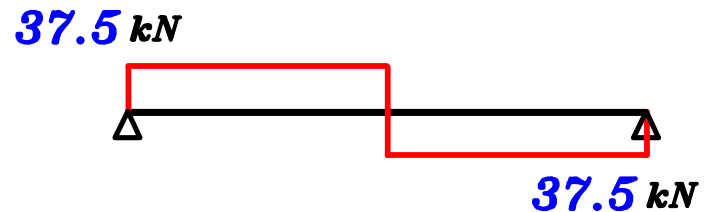


Loads on
The Beam (a b)

B.M.D. (a b)

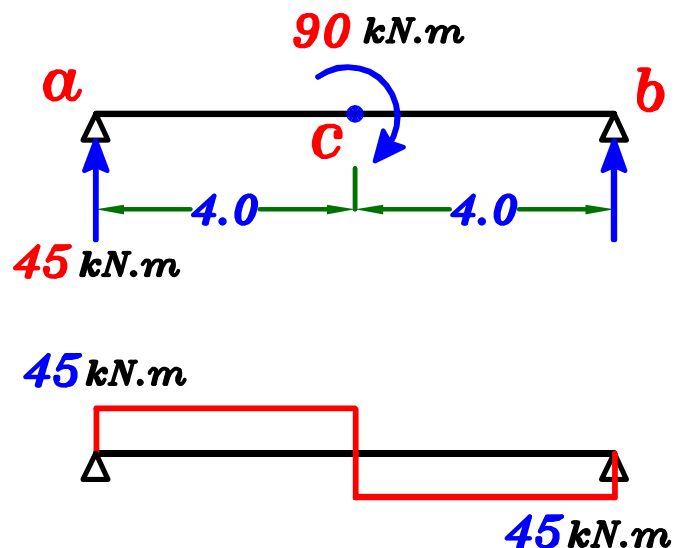


S.F.D. (a b)



Torsional Moment on
The Beam (a b)

T.M.D. (a b)



① Design of Cantilever (C d)

Sec. ①

$M_{U.L.} = 90.0 \text{ kN.m}$ R-Sec. Take $b = 250 \text{ mm}$

– Take $C_1 = 3.50 \rightarrow J = 0.78$

– Get $d = C_1 \sqrt{\frac{M_{U.L.}}{F_{cu} b}} = 3.50 \sqrt{\frac{90.0 * 10^6}{25 * 250}} = 420 \text{ mm}$

– Take $d = 450 \text{ mm}$, $t = 500 \text{ mm}$

– Get $A_s = \frac{M_{U.L.}}{J F d} = \frac{90.0 * 10^6}{0.780 * 360 * 420} = 763 \text{ mm}^2$

Check $A_{s_{min.}}$ $A_{s_{req.}} = 763 \text{ mm}^2$

$\mu_{min.} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y}\right) b d = \left(0.225 * \frac{\sqrt{25}}{360}\right) 250 * 450 = 351.5 \text{ mm}^2$

$\therefore A_{s_{req.}} > \mu_{min.} b d \therefore \text{Take } A_s = A_{s_{req.}} = 763 \text{ mm}^2$ **4 ϕ 16**

Dimensions of Beam (a b)

Take $b = 300 \text{ mm}$

* $t_{ben.} = 3.50 \sqrt{\frac{M}{F_{cu} b}} = 3.50 \sqrt{\frac{150 * 10^6}{25 * 300}} = 494.9 \text{ mm}$

* $t_{tor.} \approx \frac{3 M_t}{1.6 * b^2} = \frac{3 * 45 * 10^6}{1.6 * 300^2} = 937.5 \text{ mm}$

take $t = 950 \text{ mm}$

Design of Beam (a b) For Bending.

$$M_{U.L.} = 150 \text{ kN.m} \quad (300 * 950)$$

$$\therefore d = c_1 \sqrt{\frac{M_{U.L.}}{F_{cu} B}} \quad \therefore 900 = c_1 \sqrt{\frac{150.0 * 10^6}{25 * 300}} \rightarrow c_1 = 6.36 \rightarrow J = 0.826$$

$$\therefore A_s = \frac{M_{U.L.}}{J F_y d} = \frac{150.0 * 10^6}{0.826 * 360 * 900} = 560.4 \text{ mm}^2$$

Check $A_{s_{min.}}$ $A_{s_{req.}} = 560.4 \text{ mm}^2$

$$\mu_{min.} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y} \right) b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 300 * 900 = 843.7 \text{ mm}^2$$

$$\therefore \mu_{min.} b d > A_{s_{req.}} \xrightarrow{\text{Use}} A_{s_{min.}}$$

$$A_{s_{min.}} = 0.225 * \frac{\sqrt{F_{cu}}}{F_y} b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 300 * 900 = 843.7$$

الأقل = 728.5

$$1.3 A_{s_{req.}} = 1.3 * 560.4 = 728.5$$

الأكثر = 728.5 mm²

$$\text{st. } 360/520 \quad \frac{0.15}{100} b d = \frac{0.15}{100} * 300 * 900 = 405 \text{ mm}^2$$

Design the beam For Shear + Torsion.

$$q_u = \frac{Q}{bd} = \frac{37.5 * 10^3}{300 * 900} = 0.138 \text{ N/mm}^2$$

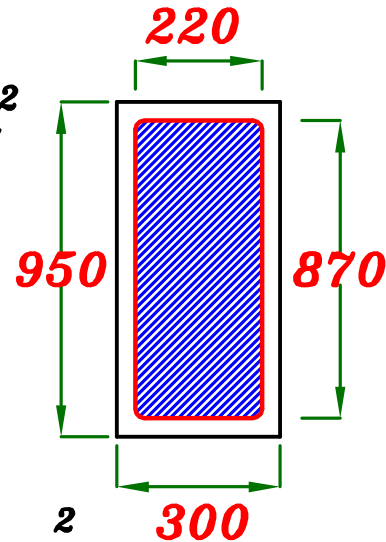
$$A_{oh} = 220 * 870 = 191400 \text{ mm}^2$$

$$A_o = 0.85 * A_{oh} = 0.85 * 191400 = 162690 \text{ mm}^2$$

$$P_h = 2 * 220 + 2 * 870 = 2180 \text{ mm}$$

$$t_e = \frac{A_{oh}}{P_h} = \frac{191400}{2180} = 87.80 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{45 * 10^6}{2 * 162690 * 87.80} = 1.575 \text{ N/mm}^2$$



$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2$$

$$q_{tmin} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245 \text{ N/mm}^2$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85 \text{ N/mm}^2$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.138^2 + 1.575^2} = 1.581 \text{ N/mm}^2 < q_{u_{max}} \therefore \text{o.k.}$$

$$q_u < q_{cu}, q_{tu} > q_{tmin} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\phi_s} \right)} \quad \therefore A_{str} = \frac{(45 \cdot 10^6) \cdot S_t}{(1.7)(191400)(240/1.15)}$$

$$\therefore S_t = 1.509 \cdot A_{str}$$

* Take $\phi 8 \rightarrow A_{str} = 50.3 \text{ mm}^2$

$$\therefore S_t = 1.509 \cdot A_{str} = 1.509 \cdot 50.3 = 75.9 \text{ mm} < 100 \text{ mm}$$

* Take $\phi 10 \rightarrow A_{str} = 78.5 \text{ mm}^2$

$$\therefore S_t = 1.509 \cdot A_{str} = 1.509 \cdot 78.5 = 118.4 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

$$\therefore \text{No. of stirrups/m} = \frac{1000}{S} = \frac{1000}{118.4} = 8.44 = 9.0$$

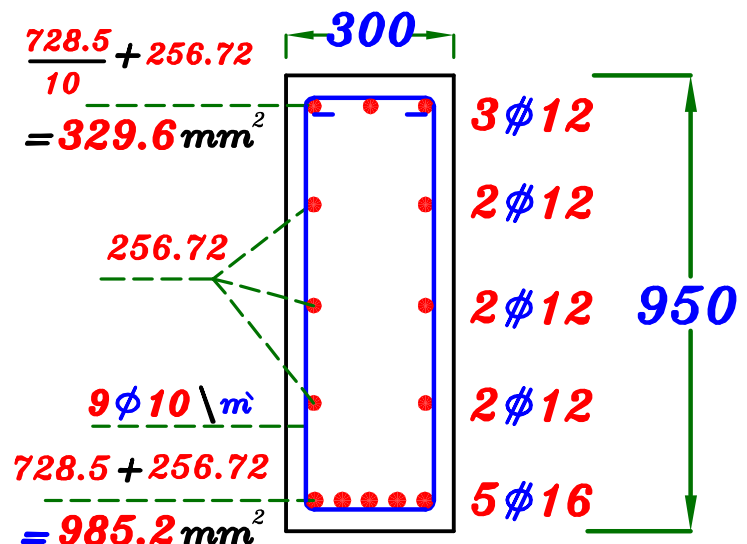
$$\therefore \text{Use Closed Stirrups } \boxed{9 \phi 10 \setminus \text{m}} \text{ 2 branches.}$$

* Longitudinal Bars. $S_t = \frac{1000}{9} = 111.1 \text{ mm}$

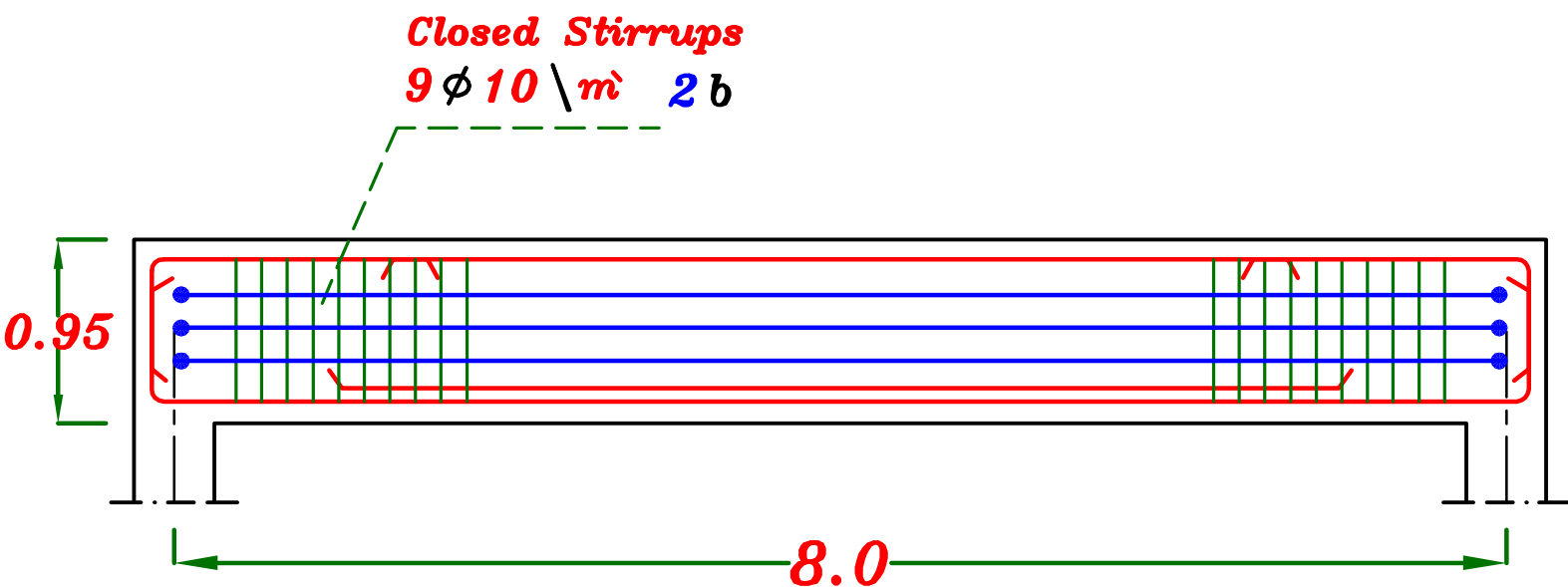
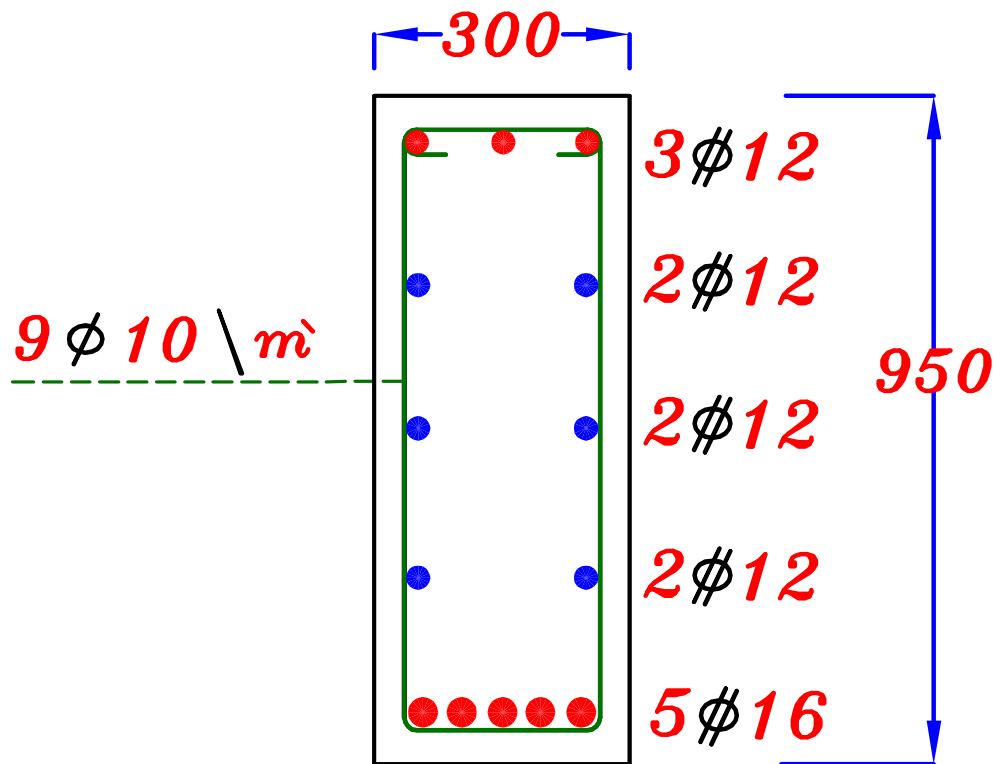
$$A_{sl} = \frac{A_{str} \cdot P_h}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = \frac{(78.5 \cdot 2180)}{111.1} \left(\frac{240}{360} \right) = 1026.9 \text{ mm}^2$$

$$\therefore \frac{A_{sl}}{4} = \frac{1026.9}{4} = 256.72 \text{ mm}^2$$

Cross Sec.
In Beam (a b)

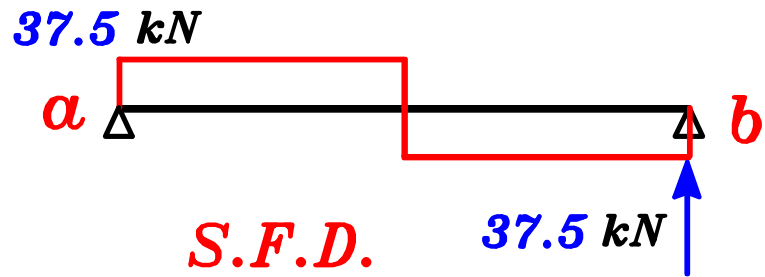


RFT. of Beam.

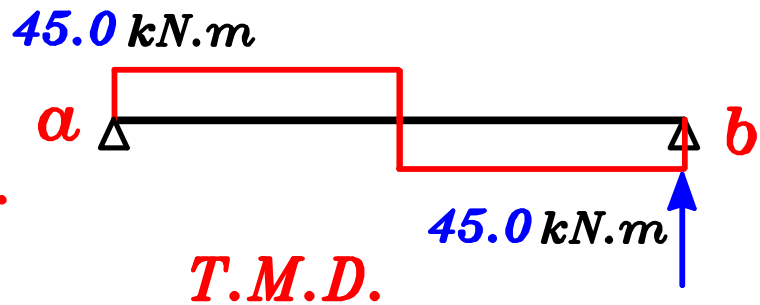


② Loads on Column (b)

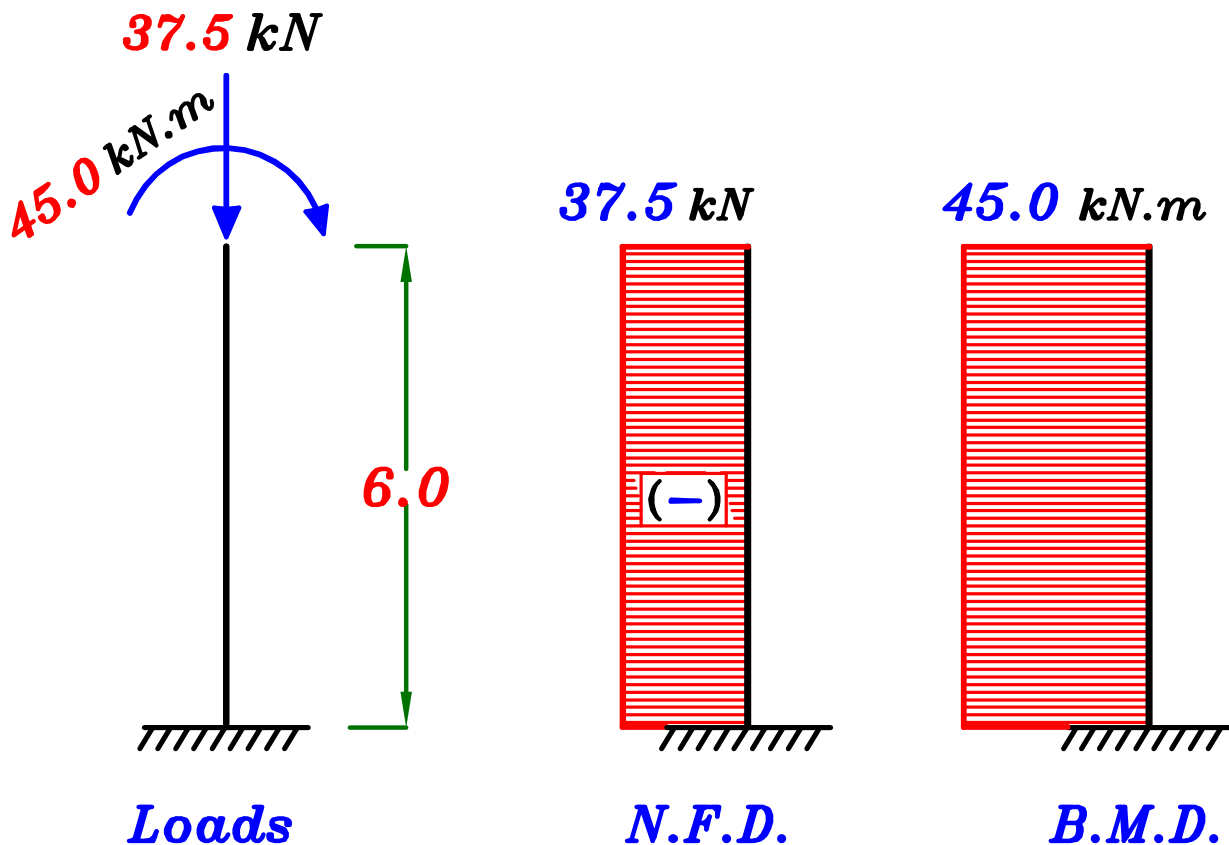
Reaction of *S.F.D.*
=
Load on the column.



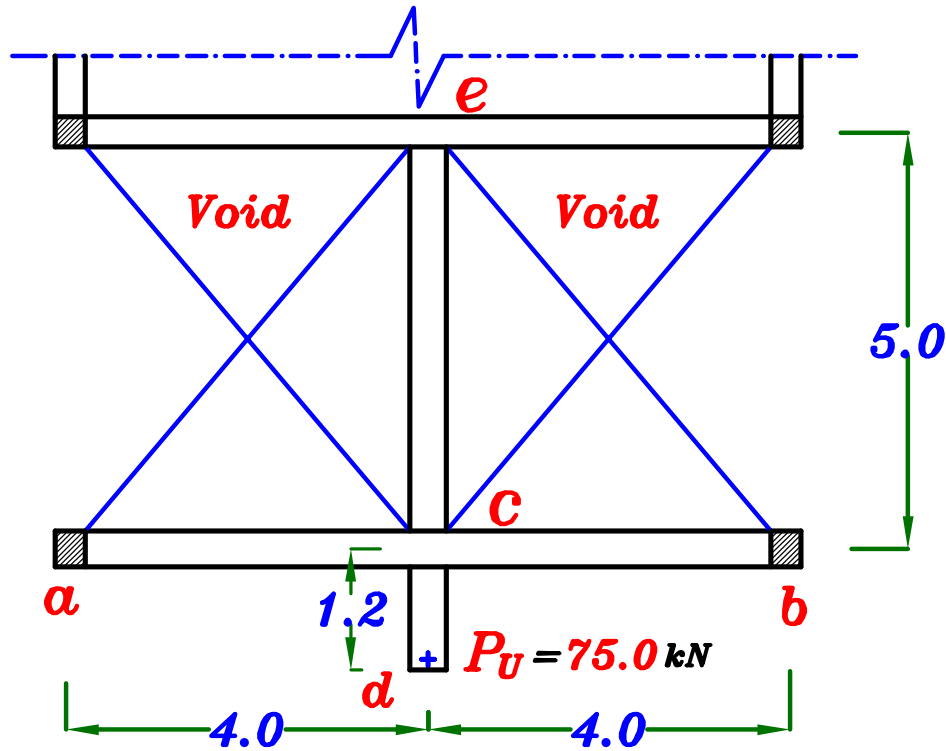
Reaction of *T.M.D.*
=
Bending on the column.



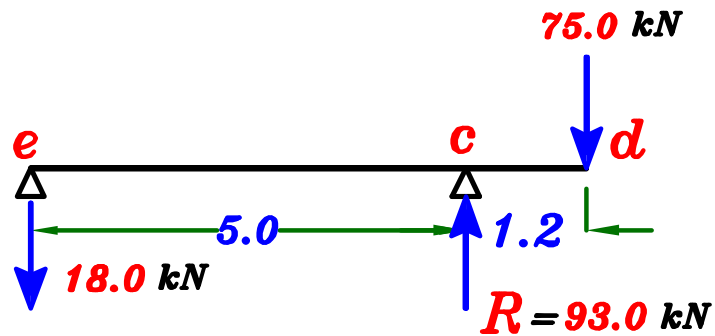
Internal Forces Diagrams on Column (b).



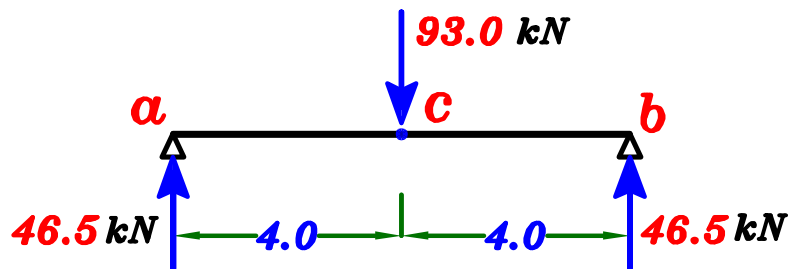
- ③ To make the Beam (a b) Free of Torsion.
We can put a member at (c e) as shown.



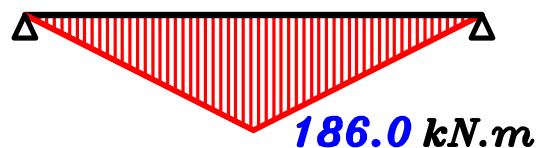
Beam (e c d)



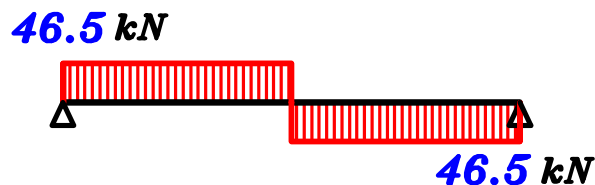
Beam (a c b)



B.M.D.



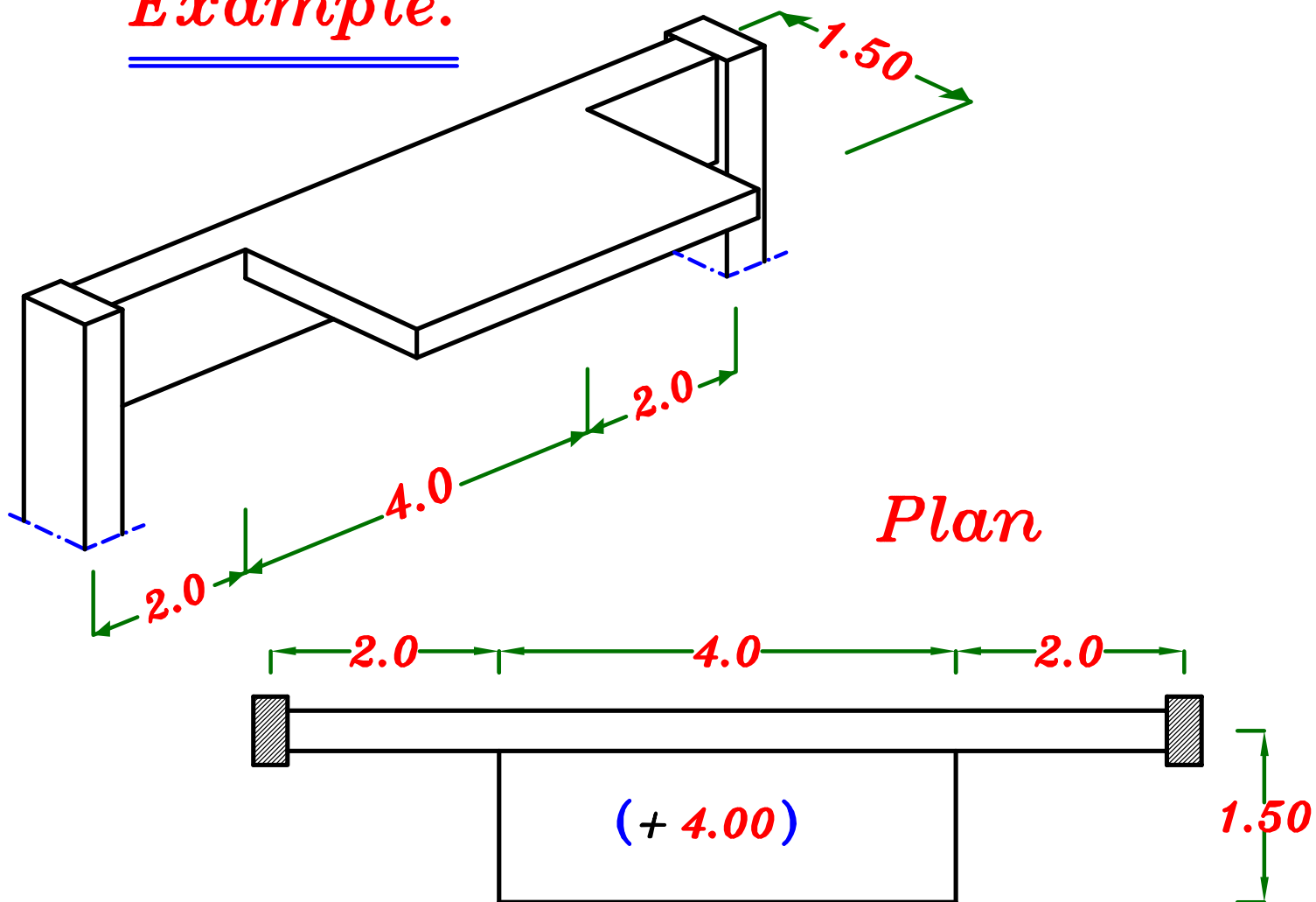
S.F.D.



T.M.D.



Example.



Data.

$$F_{cu} = 25 \text{ N/mm}^2, \quad F_y = 360 \text{ N/mm}^2$$

$$F.C. = 1.50 \text{ kN/m}^2, \quad L.L. = 2.0 \text{ kN/m}^2$$

Column (350 * 500)

Slab Level = (+ 4.00) m

Foundation Level = (- 1.50) m

Req.

- ① Complete design For The Slab , Beam & Column.
- ② Draw the details of RFT. in plan , elevation & Cross sections.

Solution.

Loads on the Slab.

$$t_s = \frac{L_c}{10} = \frac{1500}{10} = 150 \text{ mm}$$

$$t_s = 150 \text{ mm}$$

$$w_s = 1.4 (t_s \delta_c + F.C.) + 1.6 (L.L.)$$

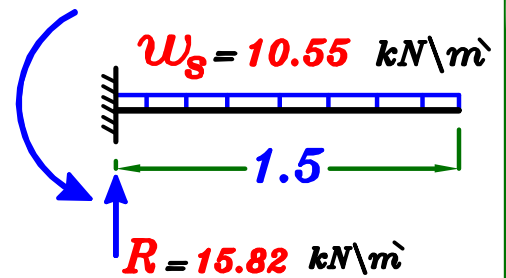
$$w_s = 1.4 (0.15 * 25 + 1.5) + 1.6 (2.0) = 10.55 \text{ kN/m}^2$$

Strip in the slab.

$$R = 10.55 * 1.5 = 15.82 \text{ kN/m}$$

$$M = \frac{10.55 * 1.5^2}{2} = 11.86 \text{ kN.m/m}$$

$$11.86 \text{ kN.m/m}$$



Design the Slab.

Sec. ①

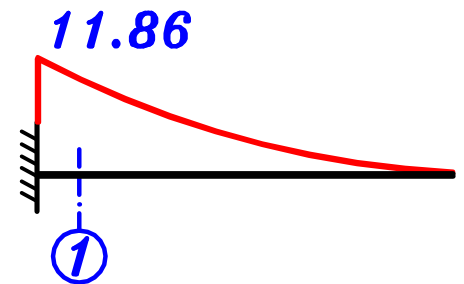
$$M_{U.L.} = 11.86 \text{ kN.m/m}$$

$$, t_s = 150 \text{ mm} , d = 130 \text{ mm}$$

$$130 = C_1 \sqrt{\frac{11.86 * 10^6}{25 * 1000}} \rightarrow C_1 = 5.96 \rightarrow J = 0.826$$

$$A_s = \frac{11.86 * 10^6}{0.826 * 360 * 130} = 306.8 \text{ mm}^2/\text{m}$$

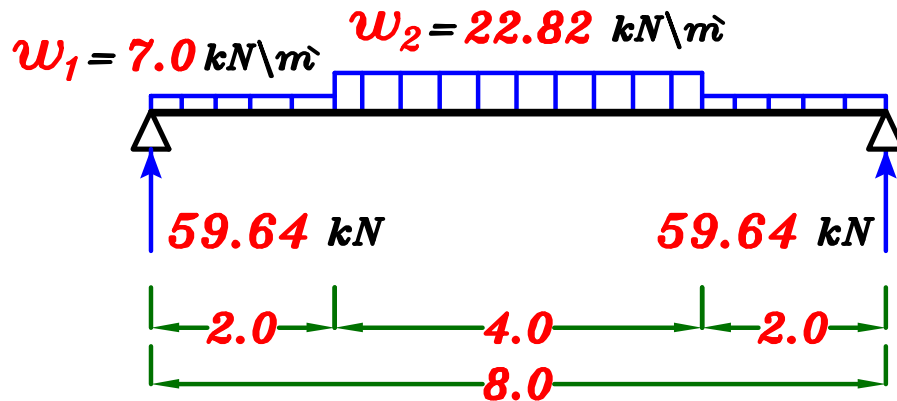
$$5 \phi 10 / \text{m}$$



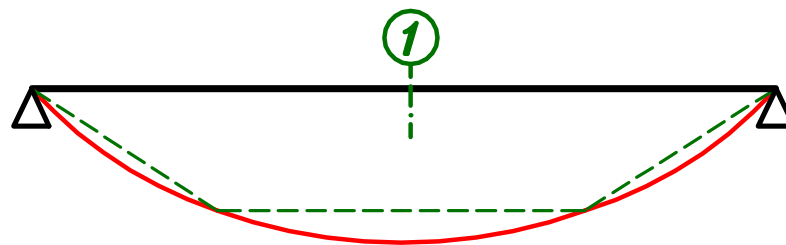
Loads on the Beam.

$$w_1 = o.w.(\text{beam}) \simeq 7.0 \text{ kN/m} \quad (U.L.)$$

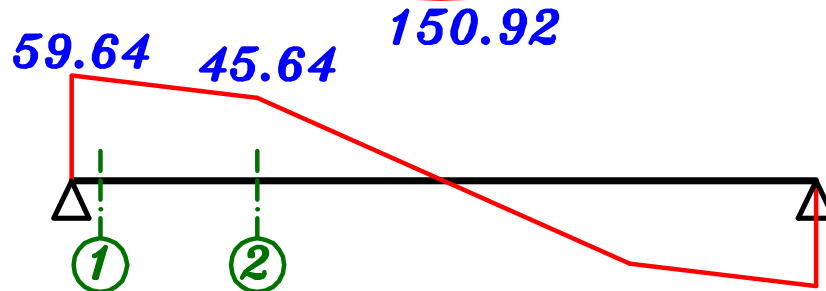
$$w_2 = o.w.(\text{beam}) + R = 7.0 + 15.82 = 22.82 \text{ kN/m}$$



B.M.D.

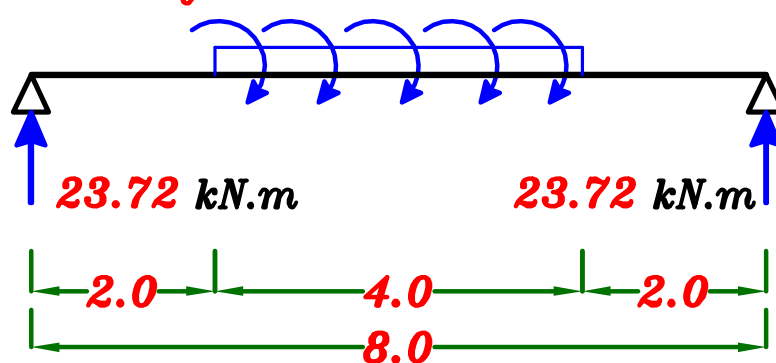


S.F.D.

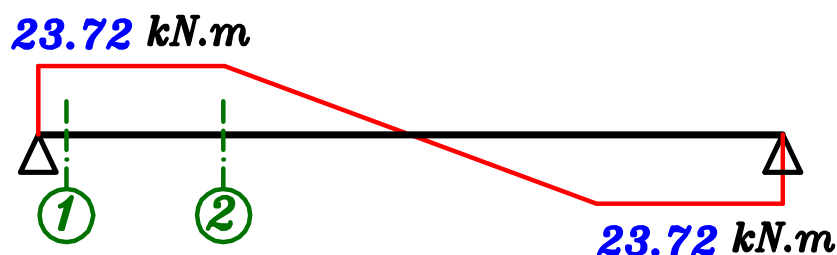


T.M.

$$m_t = 11.86 \text{ kN.m/m}$$

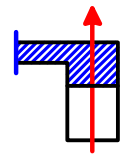


T.M.D.

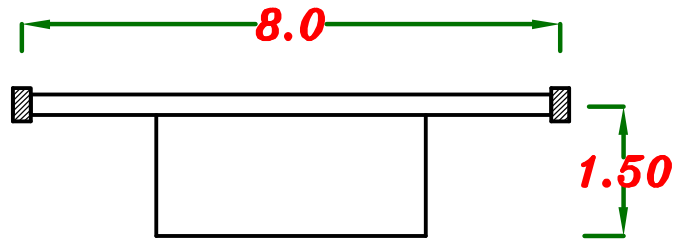
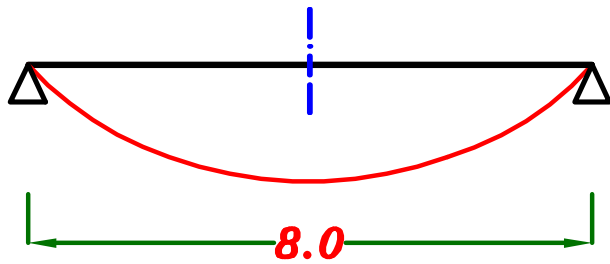


Dimensions of the Beam.

(L-sec.)



Take $b = 300$ mm



$$B = \left\{ \begin{array}{l} C.L. - C.L. = 1.5 \text{ m} = 1500 \text{ mm} \\ 6 t_s + b = 6 * 150 + 300 = 1200 \text{ mm} \\ K \frac{L}{10} + b = 1.0 * \frac{8000}{10} + 300 = 1100 \text{ mm} \end{array} \right\} \quad \boxed{B = 1100 \text{ mm}}$$

$$* t_{ben.} = C_1 \sqrt{\frac{M}{F_{cu} B}} = 6.0 \sqrt{\frac{150.92 * 10^6}{25 * 1100}} = 444.5 \text{ mm (L-sec.)}$$

$$* t_{tor.} \approx \frac{3 M_t}{1.6 * b^2} = \frac{3 * 23.72 * 10^6}{1.6 * 300^2} = 494 \text{ mm} \quad \boxed{t = 500 \text{ mm}}$$

Design the beam For bending. (300 * 500)

$$M_{U.L.} = 150.92 \text{ kN.m (L-sec.)} , \quad d = 450 \text{ mm}$$

$$450 = C_1 \sqrt{\frac{150.92 * 10^6}{25 * 1100}} \rightarrow C_1 = 6.04 \rightarrow J = 0.826$$

$$\text{- Get } A_s = \frac{M_{U.L.}}{J F_y d} = \frac{150.92 * 10^6}{0.826 * 360 * 450} = 1127.8 \text{ mm}^2$$

$$\text{Check } A_{s_{min.}} \quad A_{s_{req.}} = 1127.8 \text{ mm}^2$$

$$\mu_{min.} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y} \right) b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 250 * 450 = 351.5 \text{ mm}^2$$

$$\therefore A_{s_{req.}} > \mu_{min.} b d \therefore \text{Take } A_s = A_{s_{req.}} = 1127.8 \text{ mm}^2$$

Design the beam For Shear + Torsion.

Sec. ①

$$q_u = \frac{Q}{bd} = \frac{59.64 * 10^3}{300 * 450} = 0.441 \text{ N/mm}^2$$

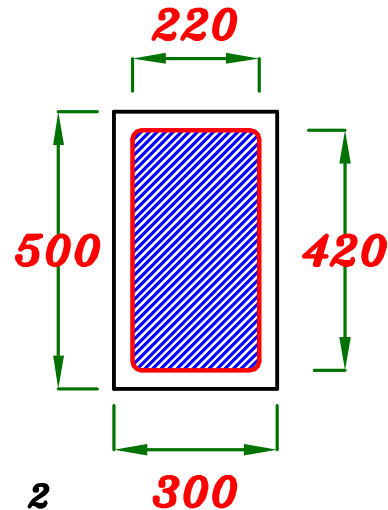
$$A_{oh} = 220 * 420 = 92400 \text{ mm}^2$$

$$A_o = 0.85 * A_{oh} = 0.85 * 92400 = 78540 \text{ mm}^2$$

$$P_h = 2 * 220 + 2 * 420 = 1280 \text{ mm}$$

$$t_e = \frac{A_{oh}}{P_h} = \frac{92400}{1280} = 72.18 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{23.72 * 10^6}{2 * 78540 * 72.18} = 2.09 \text{ N/mm}^2$$



$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2$$

$$q_{tmin} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245 \text{ N/mm}^2$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85 \text{ N/mm}^2$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.441^2 + 2.09^2} = 2.136 \text{ N/mm}^2 < q_{u_{max}} \therefore \text{o.k.}$$

$$q_u < q_{cu} , q_{tu} > q_{tmin} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\phi_s} \right)} \quad \therefore A_{str} = \frac{(23.72 \cdot 10^6) \cdot S_t}{(1.7)(92400) (240/1.15)}$$

$$\therefore \boxed{S_t = 1.382 \cdot A_{str}}$$

* Take $\phi 8 \rightarrow A_{str} = 50.3 \text{ mm}^2$

$$\therefore S_t = 1.382 \cdot A_{str} = 1.382 \cdot 50.3 = 69.51 \text{ mm} < 100 \text{ mm}$$

* Take $\phi 10 \rightarrow A_{str} = 78.5 \text{ mm}^2$

$$\therefore S_t = 1.382 \cdot A_{str} = 1.382 \cdot 78.5 = 108.48 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

$$\therefore \text{No. of stirrups/m} = \frac{1000}{S} = \frac{1000}{108.48} = 9.21 = 10$$

$$\therefore \text{Use Closed Stirrups } \boxed{10 \phi 10 \text{ m}} \text{ 2 branches.}$$

* Longitudinal Bars.

$$S_t = \frac{1000}{10} = 100 \text{ mm}$$

$$A_{sl} = \frac{A_{str} \cdot P_h}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = \frac{(78.5 \cdot 1280)}{100} \left(\frac{240}{360} \right) = 669.8 \text{ mm}^2$$

$$\therefore \frac{A_{sl}}{4} = \frac{669.8}{4} = 167.45 \text{ mm}^2$$

Check IF the Stirrups $5\phi 10\backslash m$ is safe.

Sec. ② $q_u = \frac{Q}{bd} = \frac{45.64 * 10^3}{300 * 450} = 0.338 \text{ N/mm}^2$

$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{23.72 * 10^6}{2 * 78540 * 72.18} = 2.09 \text{ N/mm}^2$

$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.338^2 + 2.09^2} = 2.11 \text{ N/mm}^2 < q_{u_{max}} \therefore \text{o.k.}$

$q_u < q_{cu} , q_{tu} > q_{tmin}$ \therefore The Stirrups will resist Torsion only.

* Check Stirrups. $\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\phi_s} \right)}$

$\therefore A_{str} = \frac{(23.72 * 10^6) * S_t}{(1.7)(92400)(360/1.15)} \therefore S_t = 2.073 * A_{str}$

* The given $\phi 10 \rightarrow A_{str} = 78.5 \text{ mm}^2$

$\therefore S_t = 2.073 * A_{str} = 2.073 * 78.5 = 162.7 \text{ mm}$

$\therefore \text{No. of stirrups required} \backslash m = \frac{1000}{S} = \frac{1000}{162.7} = 6.14 > 5.0$

\therefore the stirrups $5\phi 10\backslash m$ is not enough.

We Should increase the slab reinforcement to $7\phi 10\backslash m$

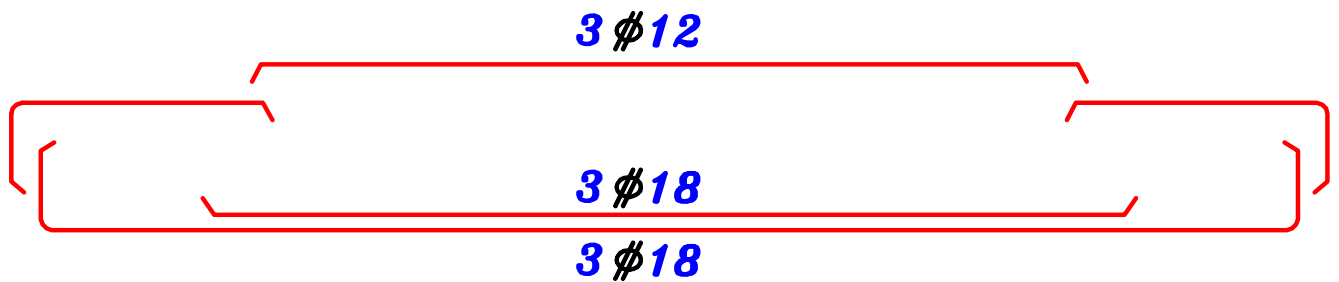
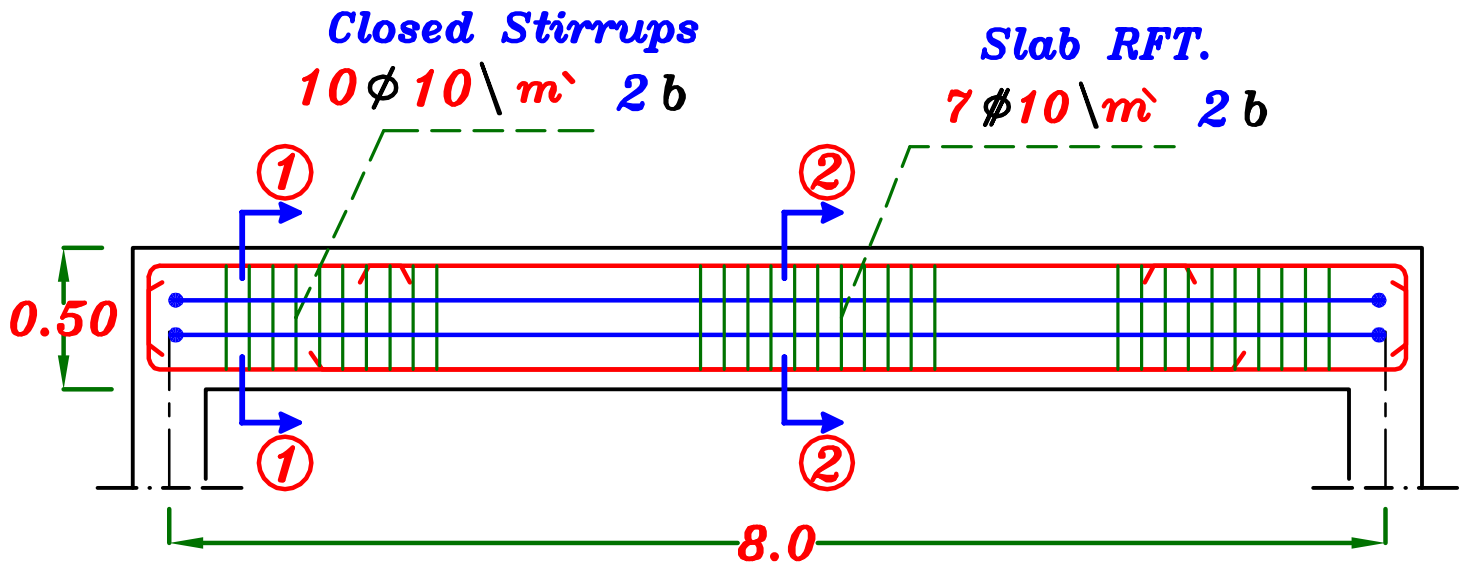
* Longitudinal Bars. $S_t = \frac{1000}{7} = 142.85 \text{ mm}$

$A_{sl} = \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = \frac{(78.5 * 1280)}{142.85} \left(\frac{360}{360} \right) = 703.39 \text{ mm}^2$

$\therefore \frac{A_{sl}}{4} = \frac{703.39}{4} = 175.84 \text{ mm}^2$

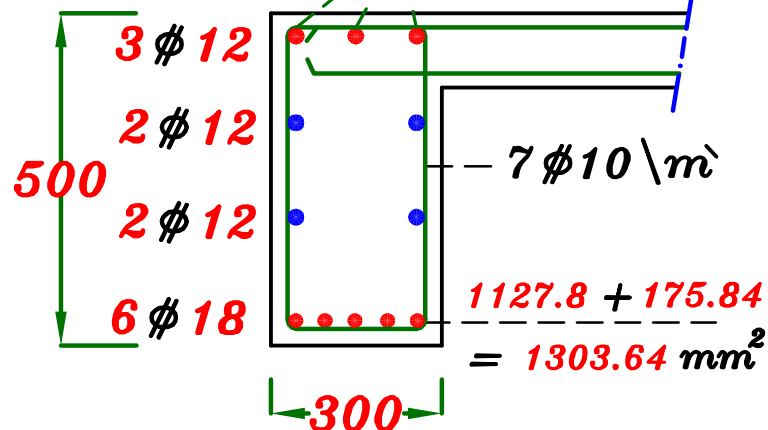
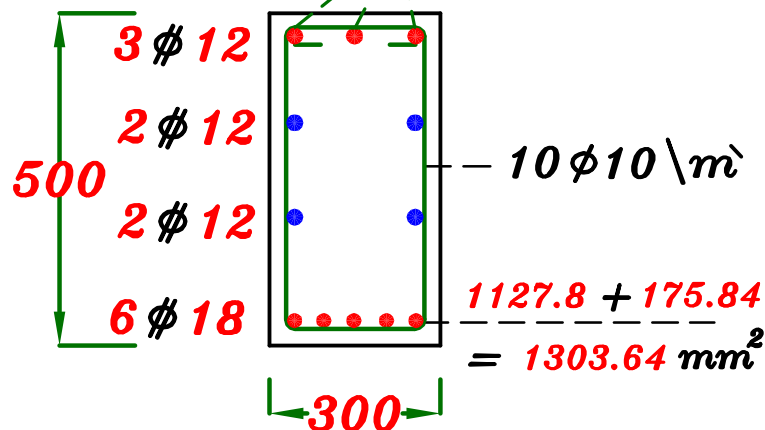
ملحوظه نأخذ قيمه A_{sl} الاكبر فى كل القطاعات التى فى الكمره .

$$\frac{A_{sl}}{4} = 175.84 \text{ mm}^2$$



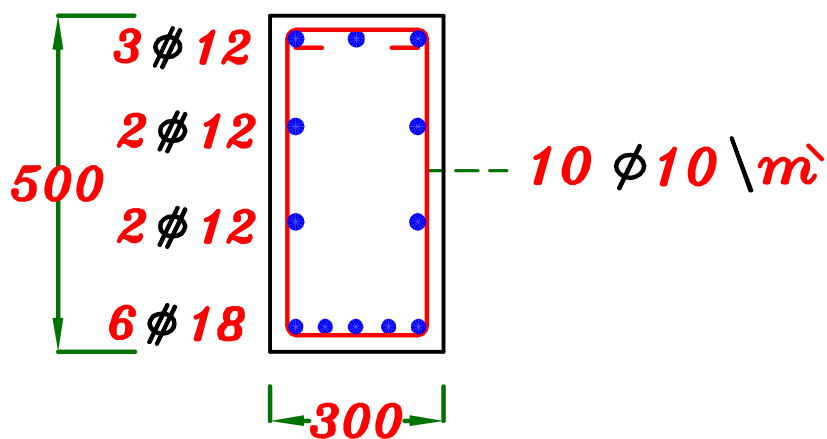
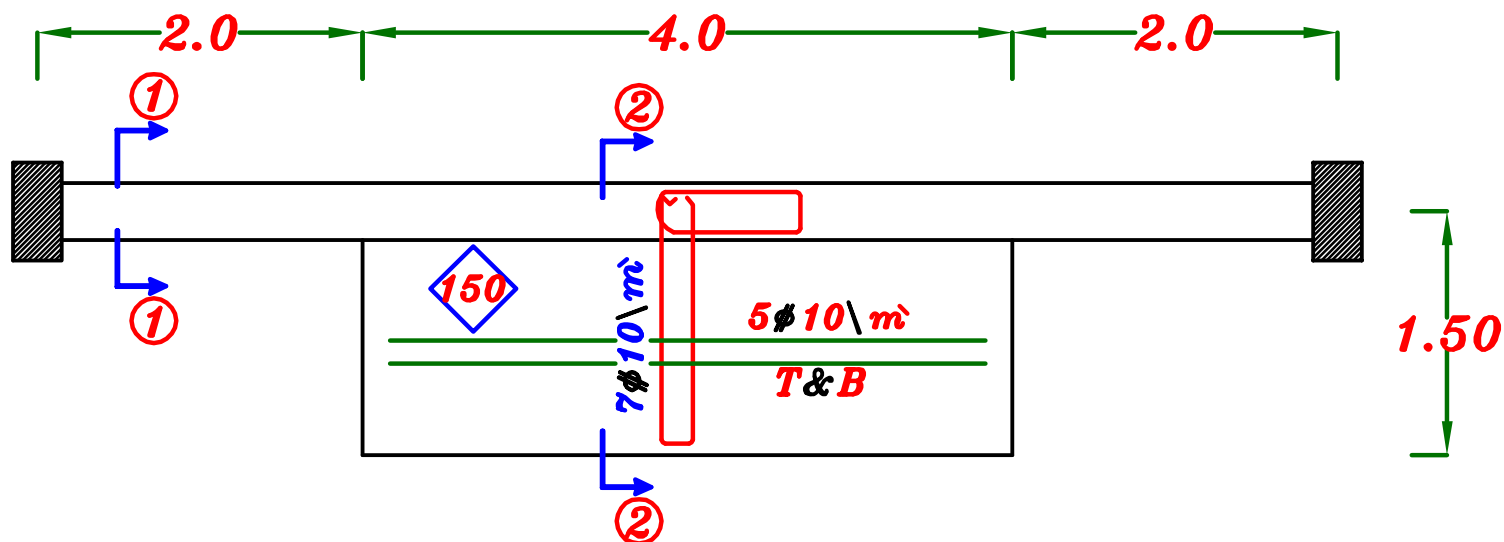
$$\frac{1127.8}{10} + 175.84 = 288.62 \text{ mm}^2$$

$$\frac{1127.8}{10} + 175.84 = 288.62 \text{ mm}^2$$

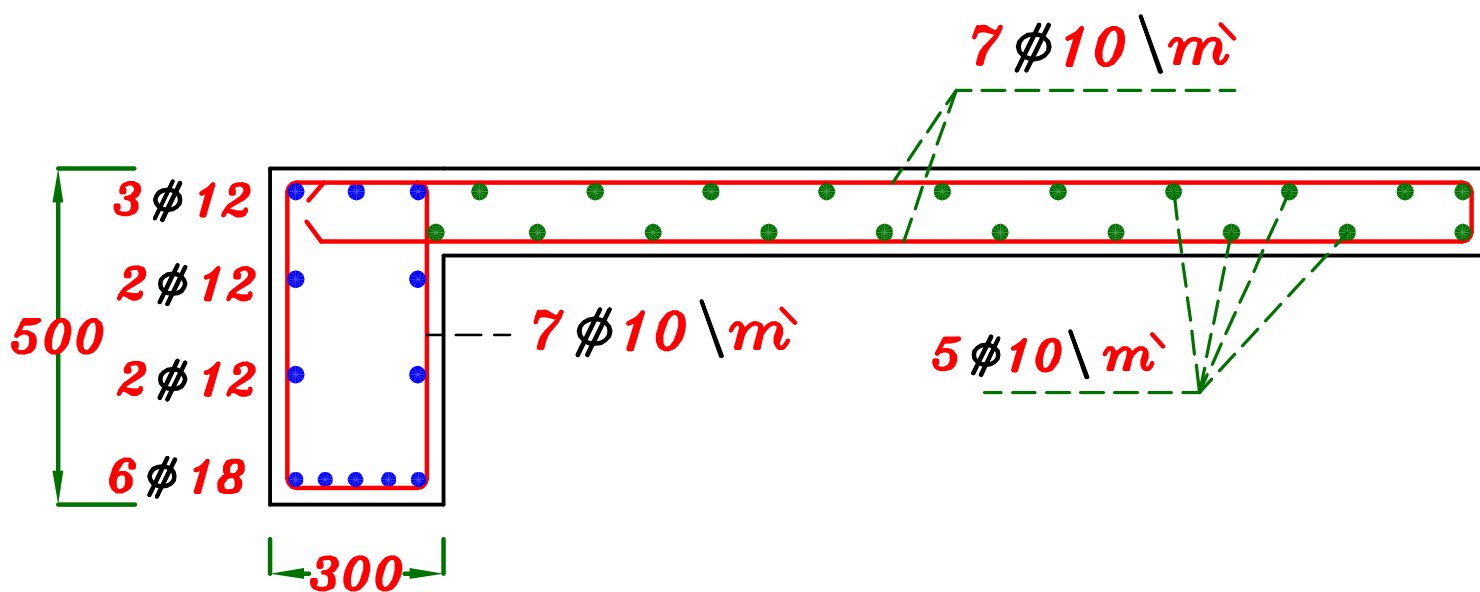


Sec. (1-1)

Sec. (2-2)

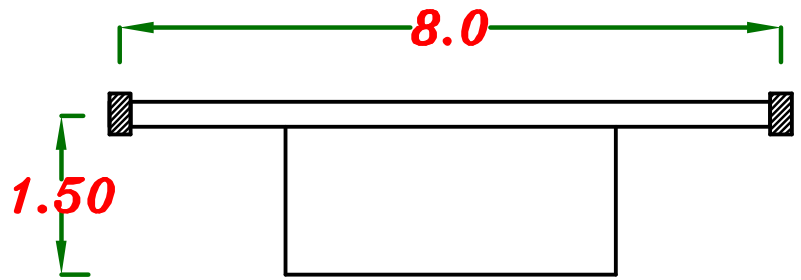


Sec. (1-1)

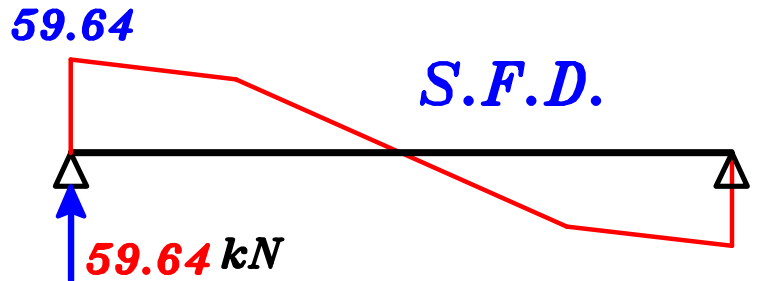


Sec. (2-2)

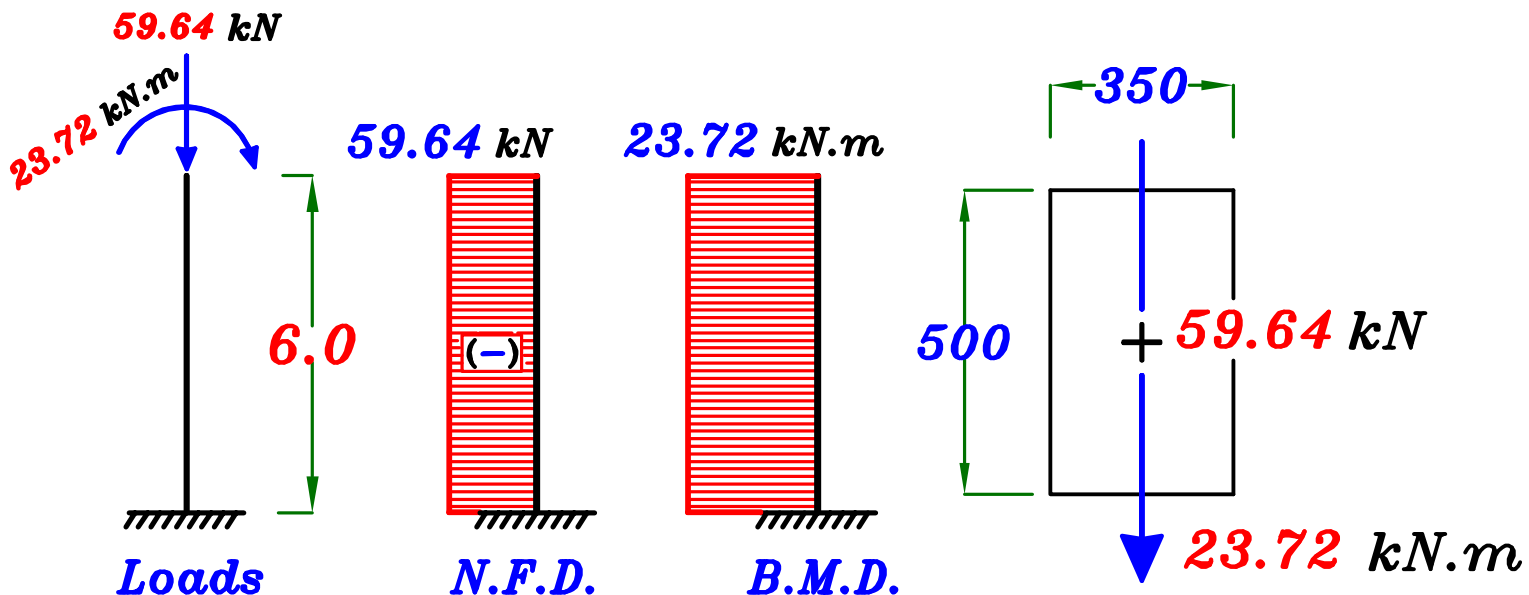
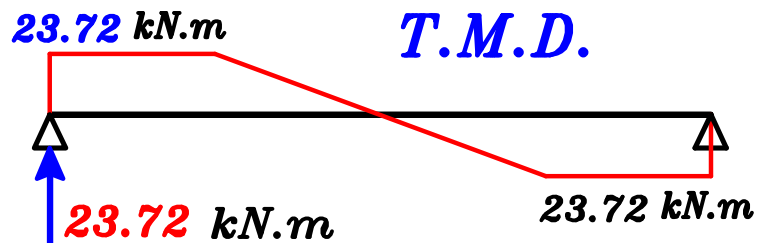
Design of the Column.



Reaction of **S.F.D.**
=
Load on the column.

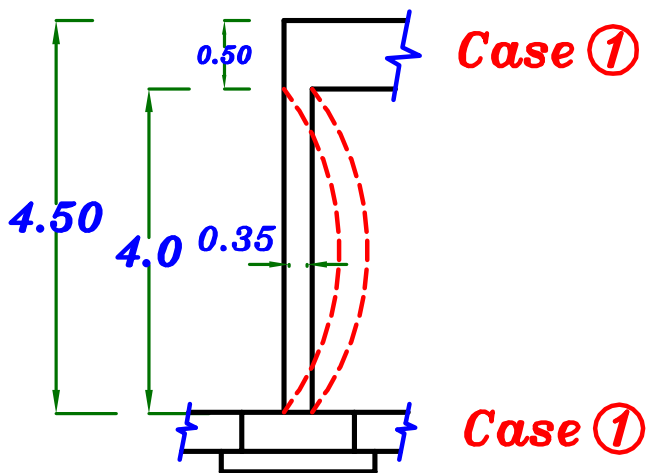


Reaction of **T.M.D.**
=
Bending on the column.



Check Buckling.

① In plane.

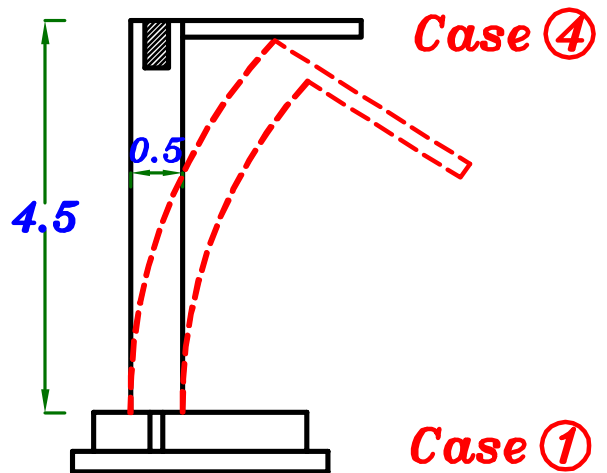


Upper Condition Case ① } $K = 1.2$
 Lower Condition Case ① }

$$H_o = 4.0 \text{ m}$$

$$\lambda_{b_{in}} = \frac{1.2 * 4.0}{0.35} = 13.7 > 10$$

② Out of plane.



Upper Condition Case ④ } $K = 2.2$
 Lower Condition Case ① }

$$H_o = 4.50 \text{ m}$$

$$\lambda_{b_{out}} = \frac{2.2 * 4.5}{0.50} = 19.8 > 10$$

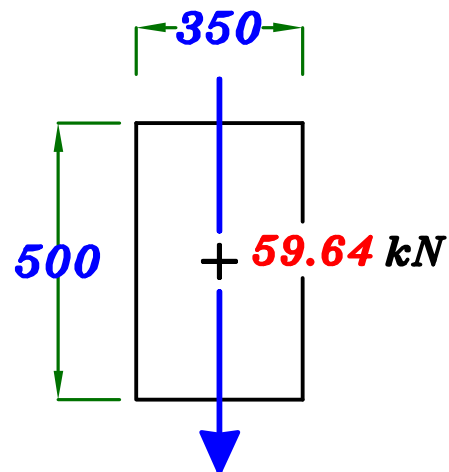
Take the bigger value of $\lambda_b = 19.8$ (Out of plane.)

$$\delta = \frac{(\lambda_b)^2 * t}{2000} = \frac{19.8^2 * 0.5}{2000} = 0.098 \text{ m}$$

$$M_{add.} = P * \delta = 59.64 * 0.098 = 5.84 \text{ kN.m}$$

$$\therefore M_{des.} = M_{ext.} + M_{add.}$$

$$\therefore M_{des.} = 23.72 + 5.84 = 29.56 \text{ kN.m}$$



$$23.72 + 5.84 = 29.56 \text{ kN.m}$$

Design the Sec.

$$e = \frac{M}{P} = \frac{29.56}{59.64} = 0.495 \text{ m} \quad \therefore \frac{e}{t} = \frac{0.495}{0.50} = 0.991 > 0.5 \xrightarrow{\text{use}} e_s$$

$$e_s = e + \frac{t}{2} - c = 0.495 + \frac{0.5}{2} - 0.05 = 0.695 \text{ m}$$

$$M_s = P * e_s = 59.64 * 0.695 = 41.45 \text{ kN.m}$$

$$\therefore 450 = C_1 \sqrt{\frac{41.45 * 10^6}{25 * 350}} \rightarrow C_1 = 6.53 \rightarrow J = 0.826$$

$$\therefore A_s = \frac{M_s}{J F_y d} - \frac{P_{u.l.}}{(F_y \setminus \delta_s)} = \frac{41.45 * 10^6}{0.826 * 360 * 450} - \frac{59.64 * 10^3}{(360 \setminus 1.15)} = 119.24 \text{ mm}^2$$

$$A_{s_{min}} = \frac{0.25 + 0.052 \lambda_{max}}{100} * b * t$$

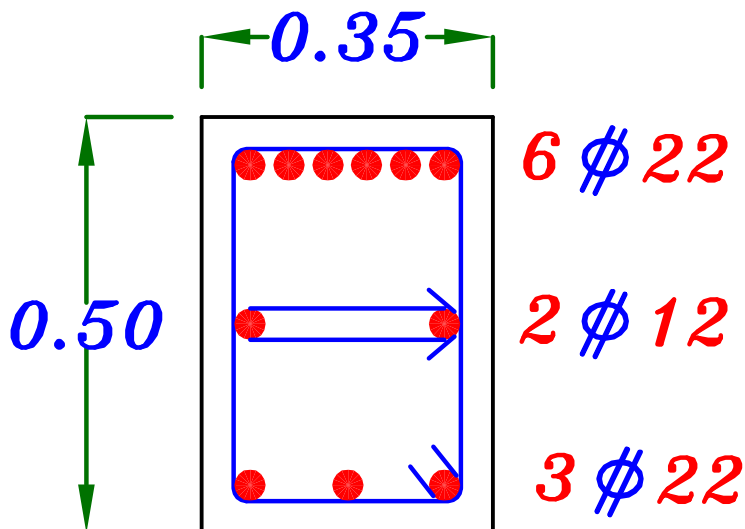
$$= \frac{0.25 + 0.052 (19.8)}{100} * 350 * 500 = 2239.3 \text{ mm}^2 > A_s$$

$$\therefore \text{Take } A_s = A_{s_{min}} = 2239.3 \text{ mm}^2 \quad (6 \phi 22)$$

$$n = \frac{b - 25}{\phi + 25} = \frac{350 - 25}{22 + 25} = 6.91 = 6.0 \text{ bars}$$

$$\text{Stirrup Hangers} = 0.4 A_s = 0.4 * 2239.3 = 895.7 \text{ mm}^2$$

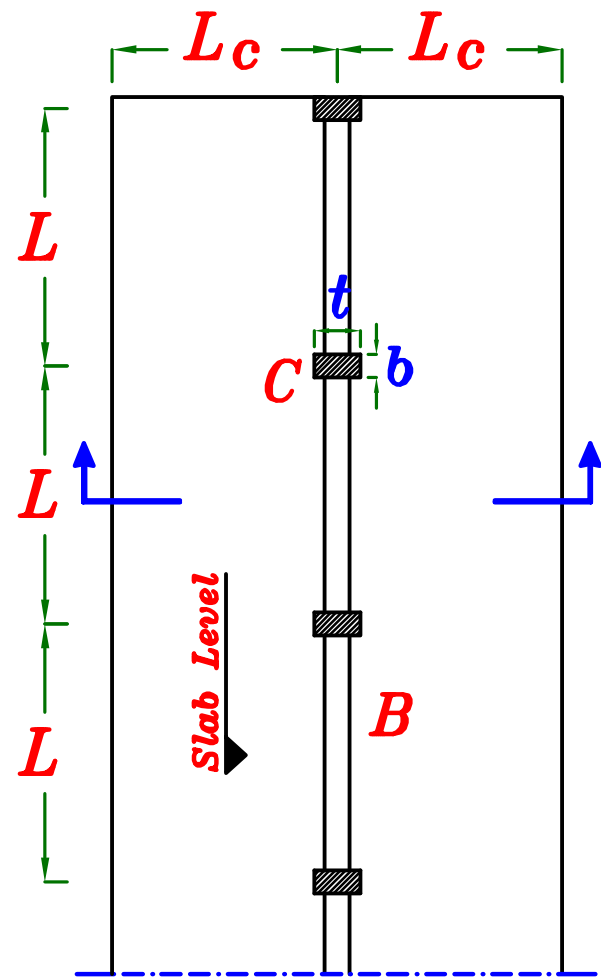
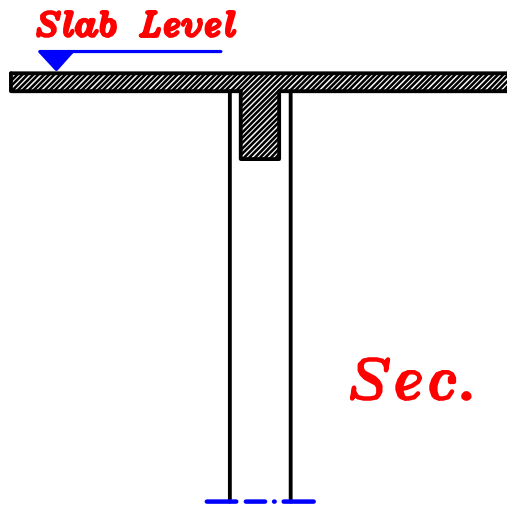
$$(3 \phi 22)$$



Example.

1- Design the beam *B*

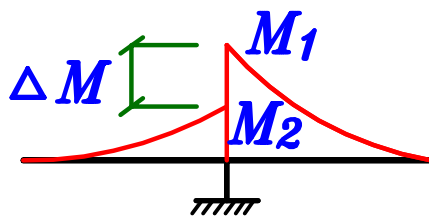
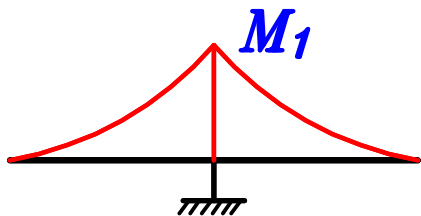
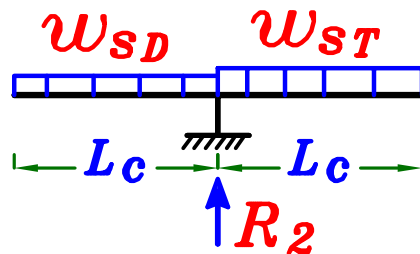
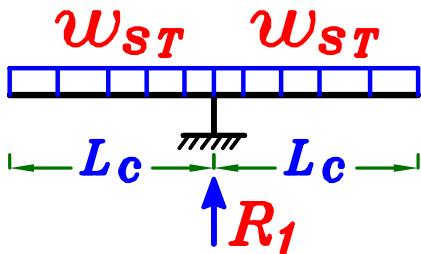
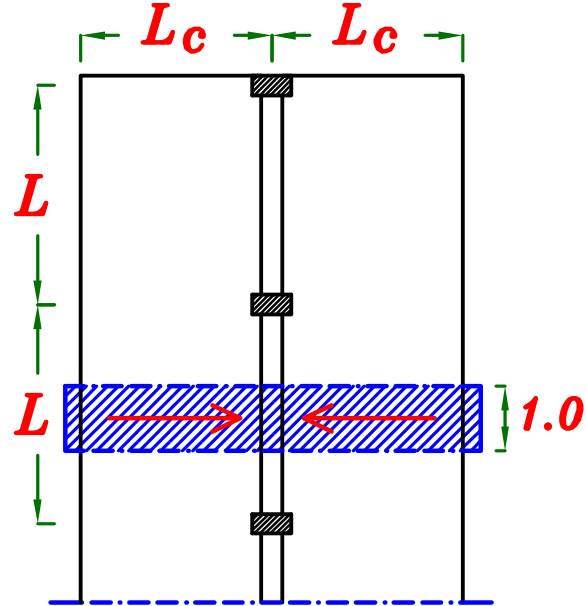
2- Design the column *C*



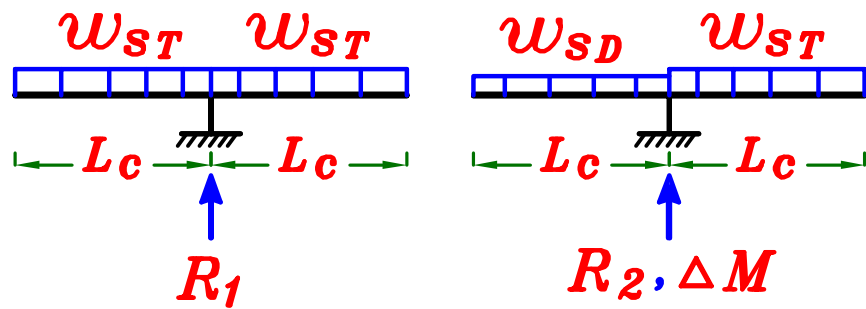
By taking strip in the slab.

$$w_{ST} = 1.4 (t_s \delta_c + F.C.) + 1.6 (L.L.)$$

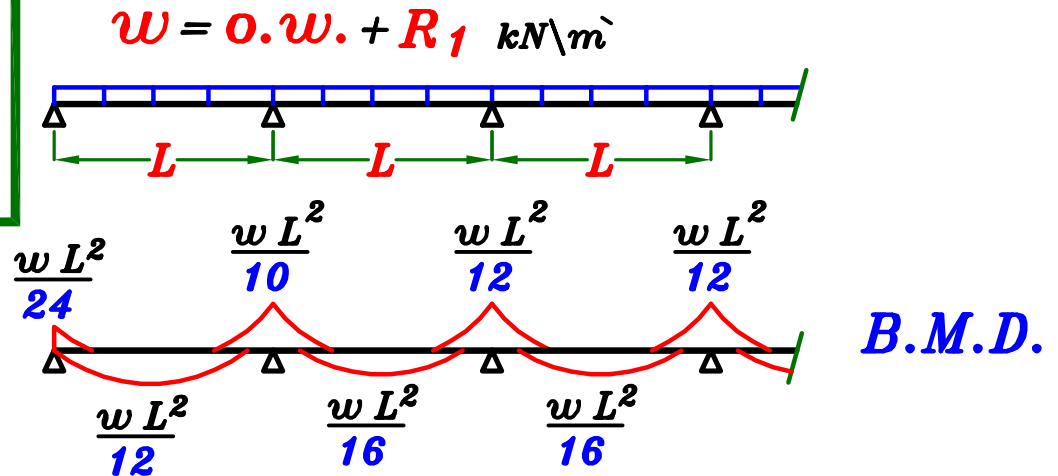
$$w_{SD} = 0.9 (t_s \delta_c + F.C.)$$



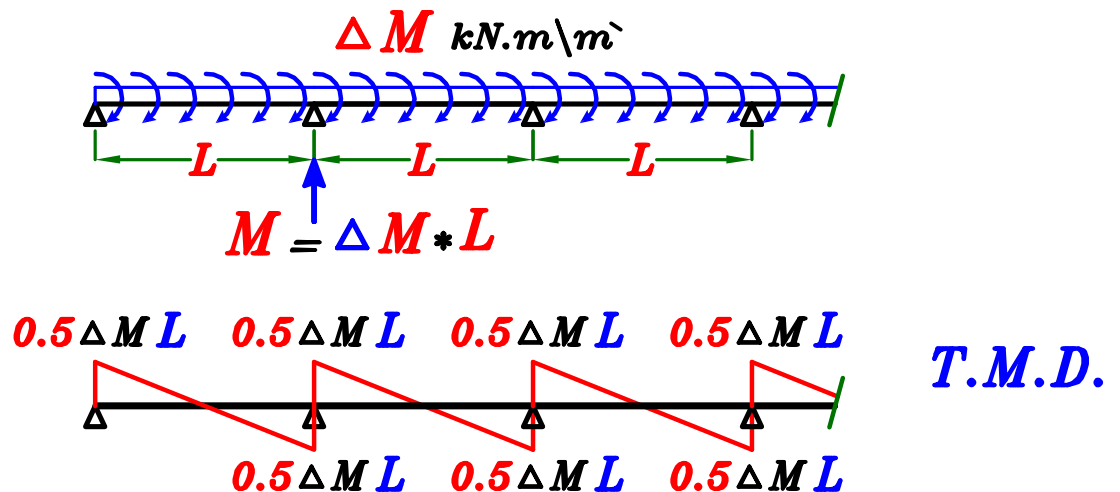
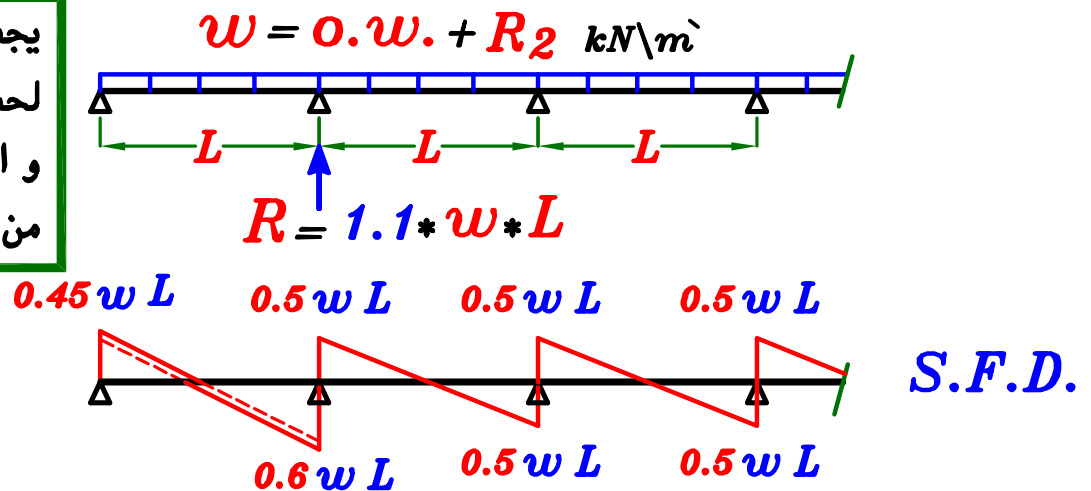
Loads on Beam B



عند حساب ال **B.M.**
للكمره نأخذ قيمه **R₁**
لنحدد اكبر **B.M.**



يجب ان تؤخذ الاحمال
لحساب ال **Shear**
وال **Torsion**
من نفس حاله التحميل



Column C

$$\begin{aligned} \text{VL. Load on Column.} &= \text{Reaction From Loads} \\ &= 1.1 * w * L = 1.1 * (o.w. + R_2) * L \end{aligned}$$

$$\begin{aligned} \text{Bending Moment on Column} &= \text{Reaction From Torsion.} \\ &= M_{ext.} = \Delta M * L \end{aligned}$$



Calculate $M_{add.}$ From Buckling

Design the Sec.

IF $M_{add.}$ In Plane

IF $M_{add.}$ Out of Plane



Example.

Data.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

$$F.C. = 2.5 \text{ kN/m}^2$$

$$L.L. = 1.0 \text{ kN/m}^2 \text{ (HL. Projection)}$$

$$\text{Spacing} = 5.0 \text{ m}$$

Req.

- ① Complete design of the slab, beam & the column.
- ② Draw the details of RFT in plan & cross-sections.

Solution.

$$\text{For } (S_1) \quad t_s = \frac{L_c}{10} = \frac{2200}{10} = 220 \text{ mm}$$

$$t_{s1} = 220 \text{ mm}$$

$$\text{For } (S_2) \quad t_s = \frac{L_c}{10} = \frac{1100}{10} = 110 \text{ mm}$$

$$t_{s2} = 120 \text{ mm}$$

Case of Total Load. (T.L.)

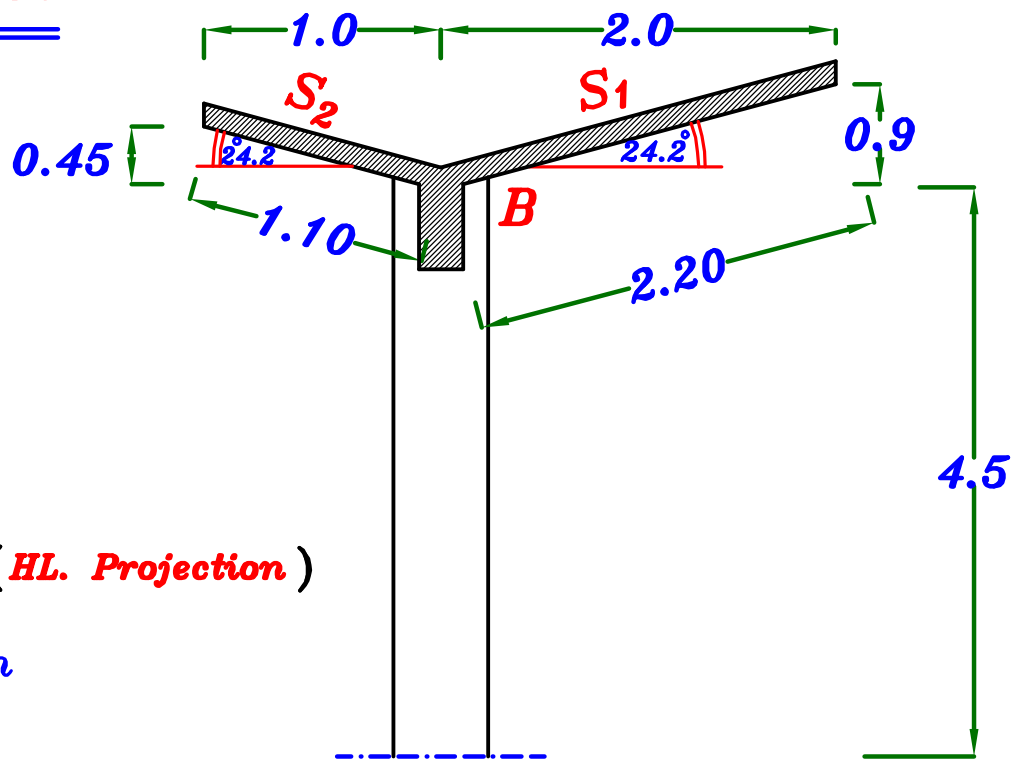
$$(w_{s1}) \quad T = 1.4 (0.22 * 25 + 2.5) + 1.6 (1.0) \cos 24.2^\circ = 12.65 \text{ kN/m}^2$$

$$(w_{s2}) \quad T = 1.4 (0.12 * 25 + 2.5) + 1.6 (1.0) \cos 24.2^\circ = 9.15 \text{ kN/m}^2$$

Case of Dead Load. (D.L.)

$$(w_{s1}) \quad D = 0.9 (0.22 * 25 + 2.5) = 7.20 \text{ kN/m}^2$$

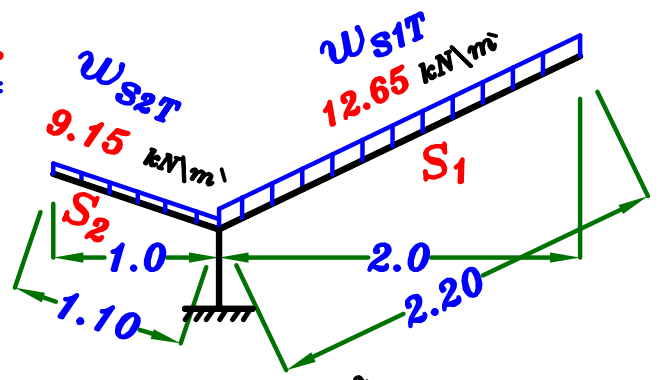
$$(w_{s2}) \quad D = 0.9 (0.12 * 25 + 2.5) = 4.95 \text{ kN/m}^2$$



Loads to design the Slabs.

$$M_1 = 12.65 * 2.20 * \frac{2.0}{2} = 27.83 \text{ kN.m/m}$$

$$M_2 = 9.15 * 1.10 * \frac{1.0}{2} = 5.03 \text{ kN.m/m}$$



Design of Slab. (S1)

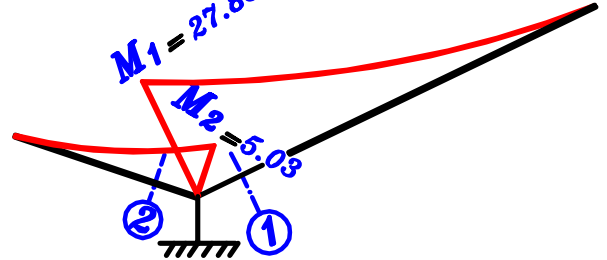
Sec. ①

$$M_{U.L.} = 27.83 \text{ kN.m/m}$$

$$, t_s = 220 \text{ mm} , d = 200 \text{ mm}$$

$$200 = C_1 \sqrt{\frac{27.83 * 10^6}{25 * 1000}} \rightarrow C_1 = 5.99 \rightarrow J = 0.826$$

$$A_s = \frac{27.83 * 10^6}{0.826 * 360 * 200} = 467.9 \text{ mm}^2/\text{m} \quad \textcircled{5 \phi 12 \backslash \text{m}}$$



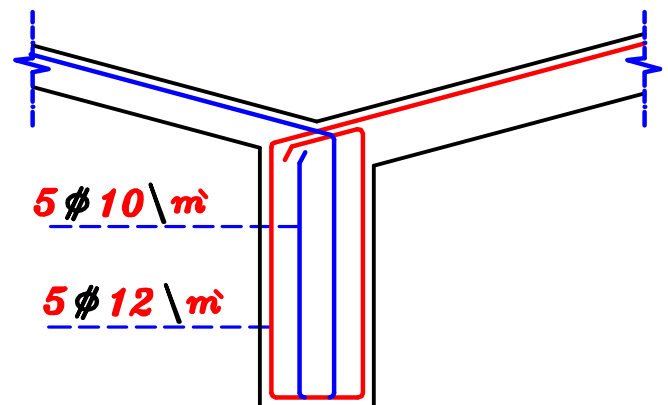
Design of Slab. (S2)

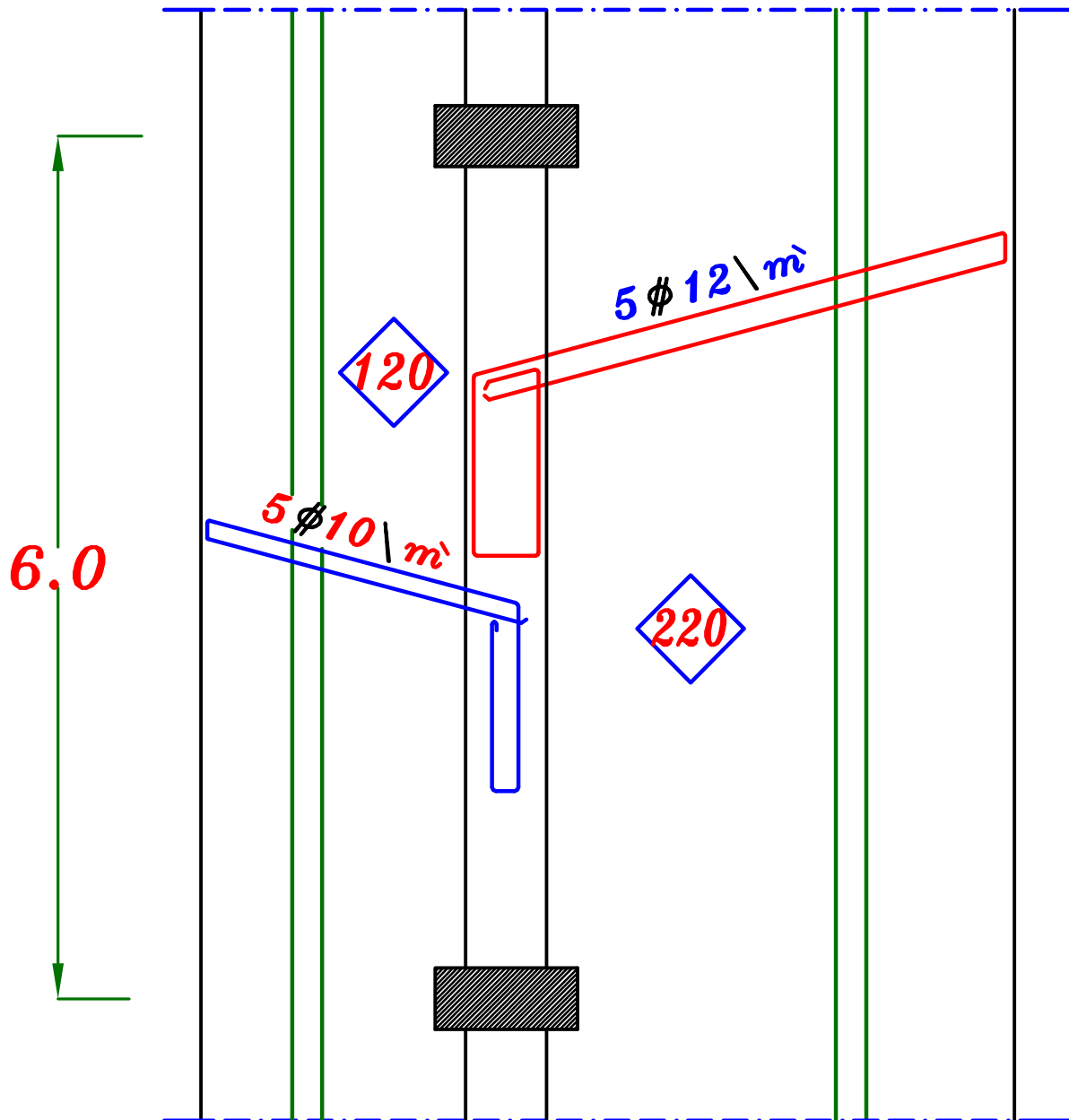
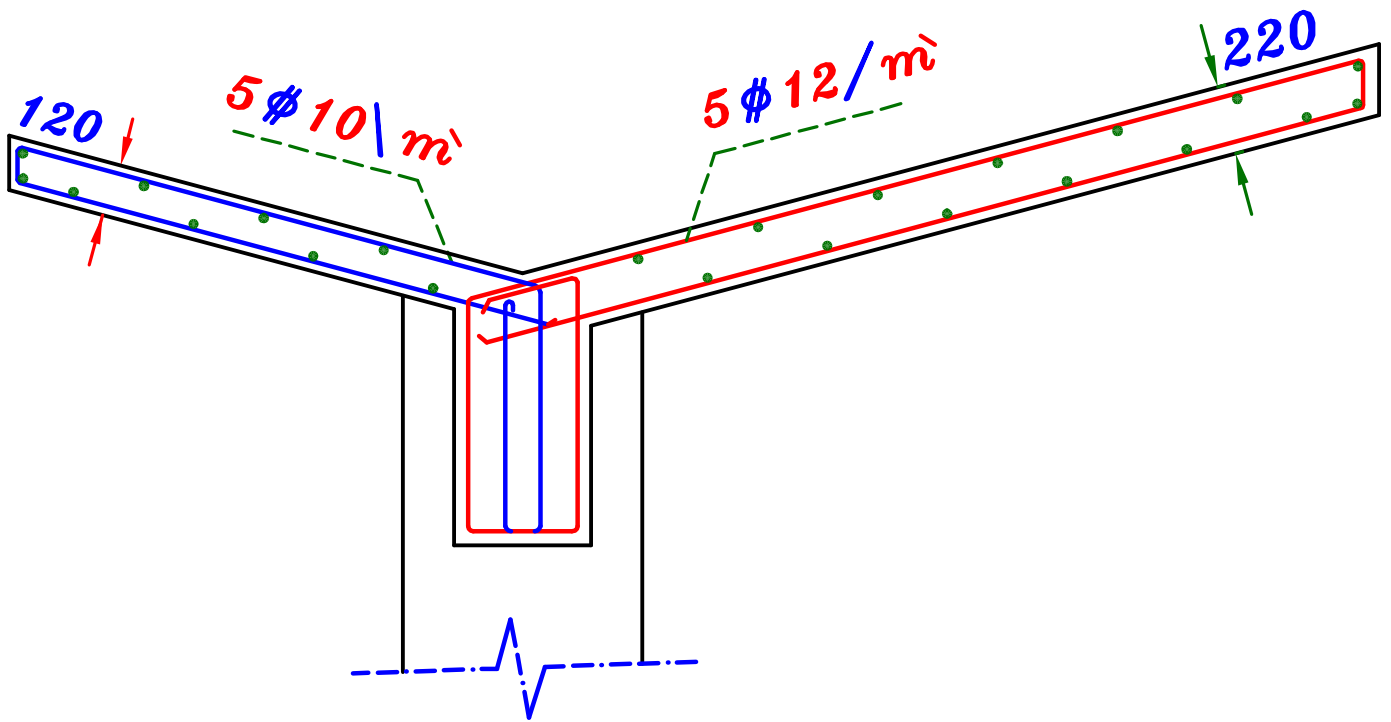
Sec. ②

$$M_{U.L.} = 5.03 \text{ kN.m/m} , t_s = 120 \text{ mm} , d = 100 \text{ mm}$$

$$100 = C_1 \sqrt{\frac{5.03 * 10^6}{25 * 1000}} \rightarrow C_1 = 7.05 \rightarrow J = 0.826$$

$$A_s = \frac{5.03 * 10^6}{0.826 * 360 * 100} = 169.1 \text{ mm}^2/\text{m} \quad \textcircled{5 \phi 10 \backslash \text{m}}$$



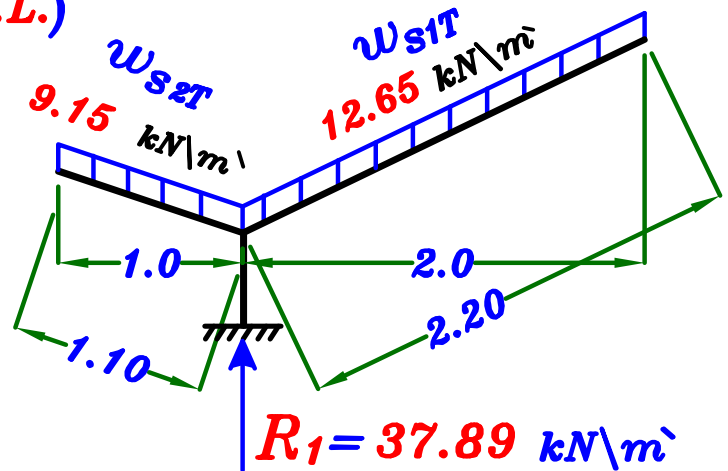


Design of the beam.

1- Draw B.M.D. , S.F.D. & T.M.D.

To Draw B.M.D. For the Beam.

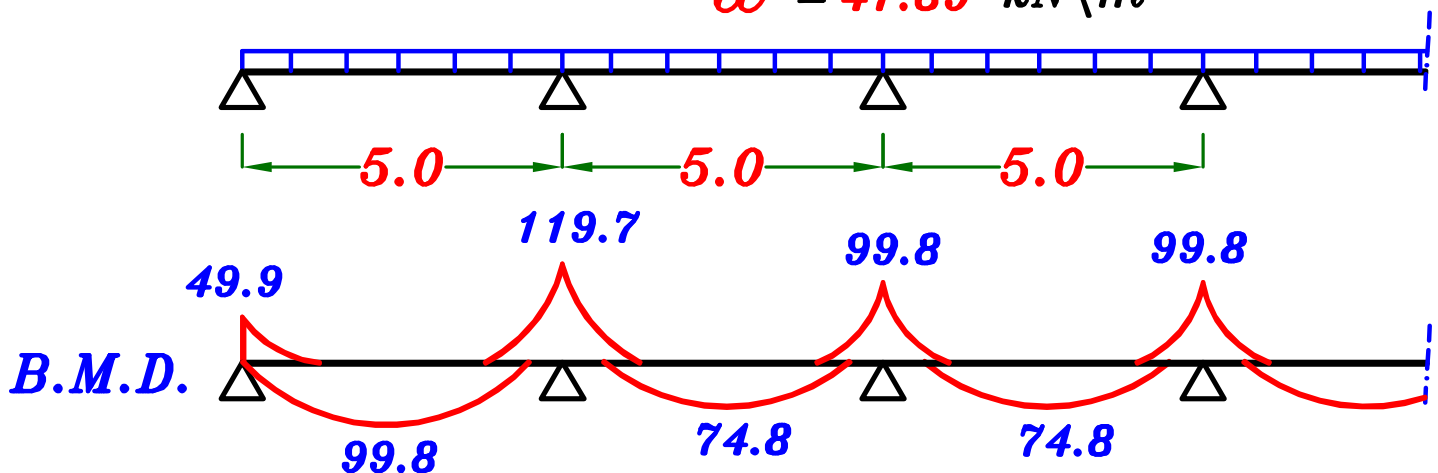
$$o.w._{(beam)} \approx 10 \text{ kN/m} \quad (U.L.)$$



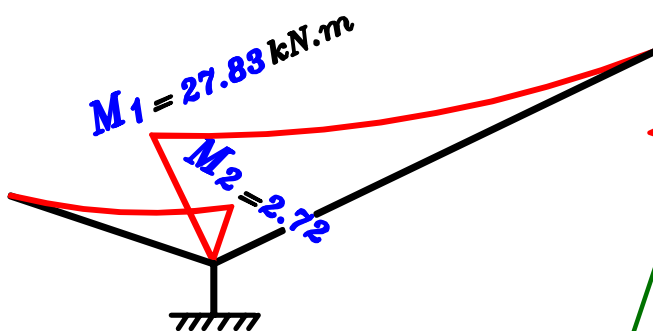
$$W = o.w._{(beam)} + R_1$$

$$= 10 + 37.89 = 47.89 \text{ kN/m}$$

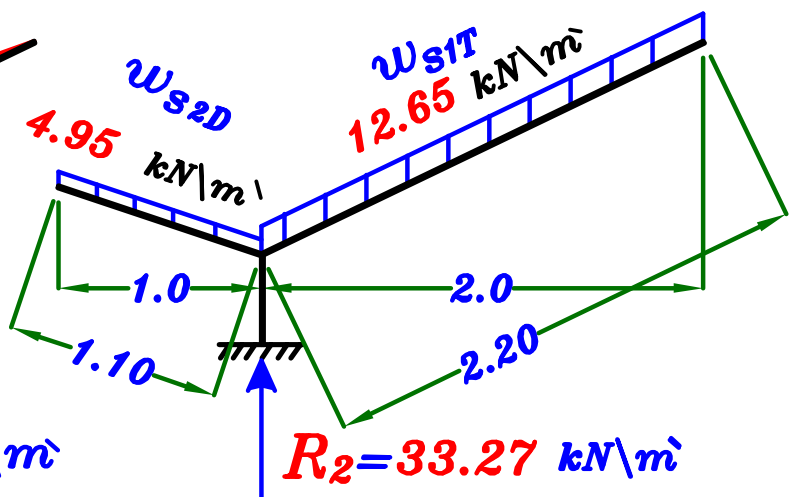
$$W = 47.89 \text{ kN/m}$$



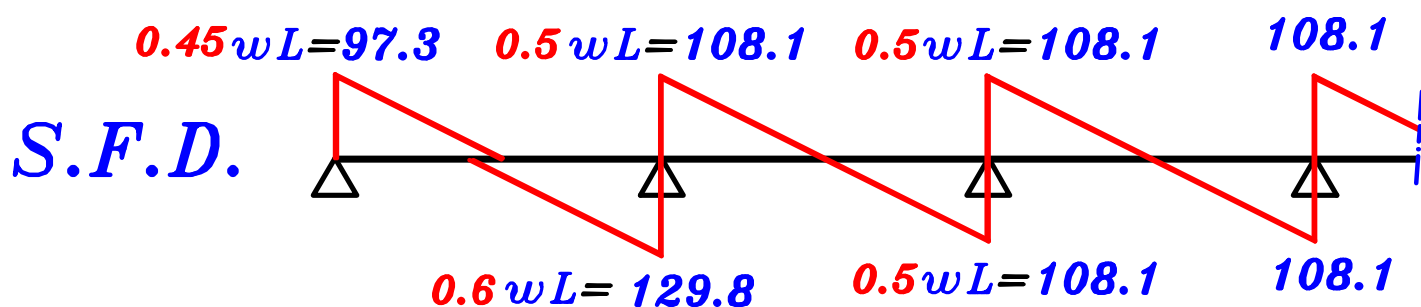
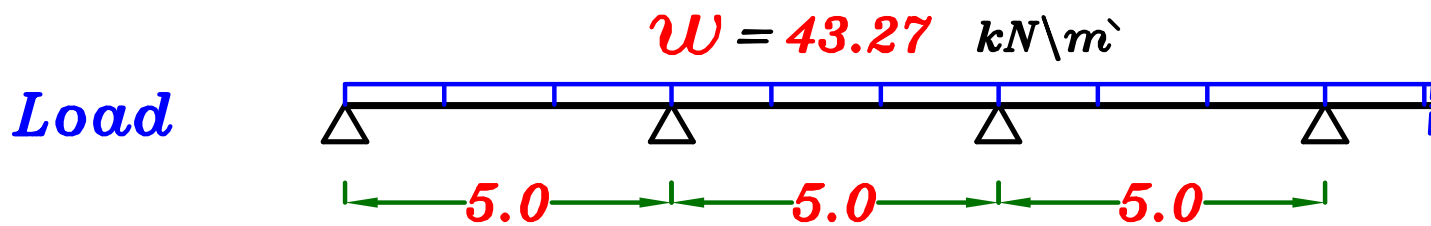
To Draw S.F.D. & T.M.D.. For the Beam.



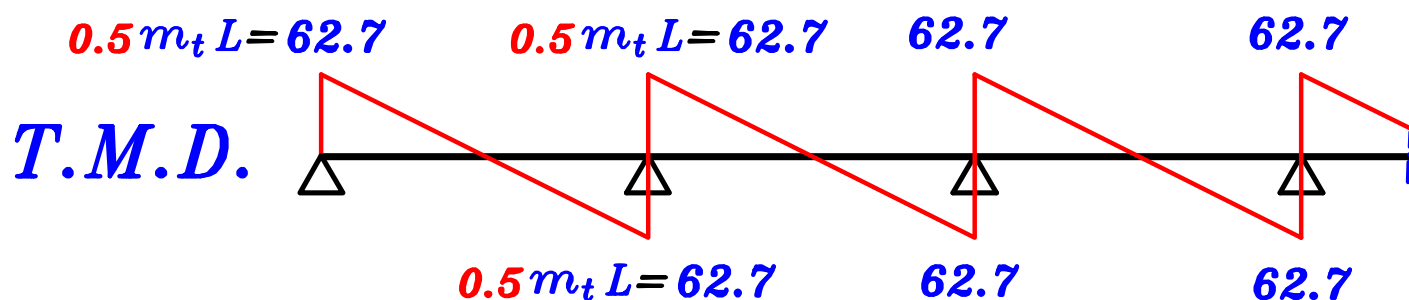
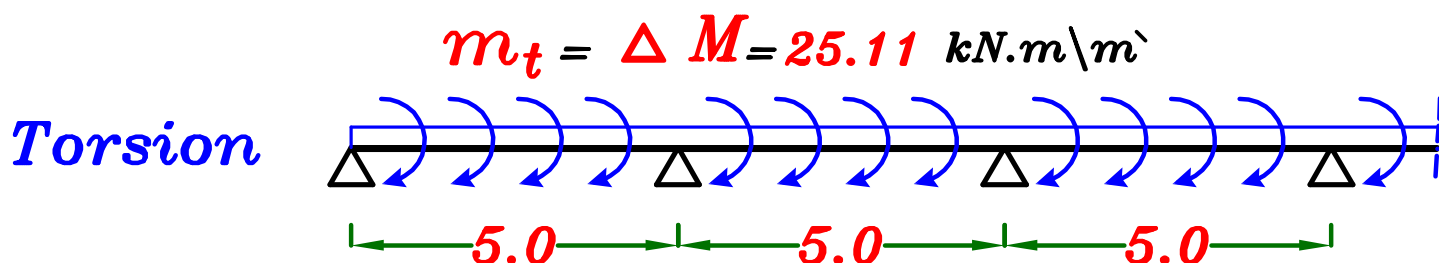
$$\Delta M = 25.11 \text{ kN.m/m}$$



$$w = 0.w_{(beam)} + R_2 = 10 + 33.27 = 43.27 \text{ kN/m}$$

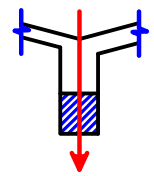


$$\Delta M = M_1 - M_2 = 27.83 - 2.72 = 25.11 \text{ kN.m/m}$$



Dimensions of the Beam.

(R-sec.)

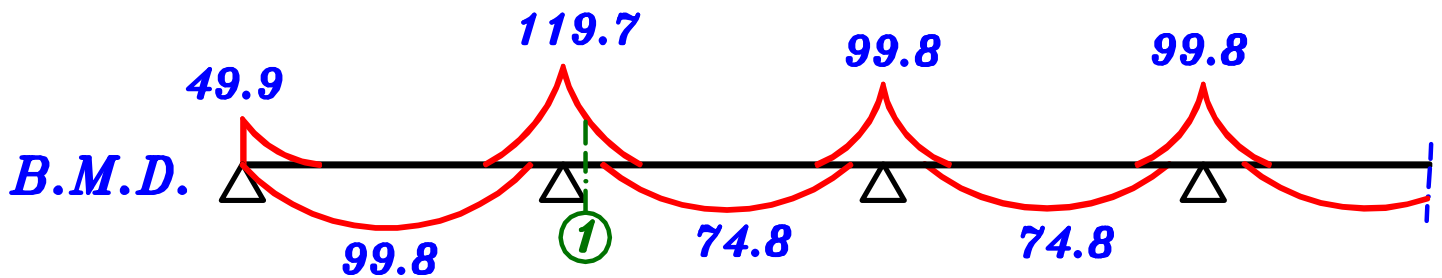


Take $b = 400 \text{ mm}$

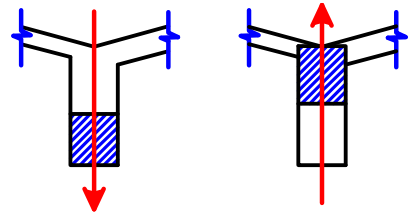
$$* t_{ben.} = C_1 \sqrt{\frac{M}{F_{cu} B}} = 3.5 \sqrt{\frac{119.7 * 10^6}{25 * 400}} = 382.9 \text{ mm (R-sec.)}$$

$$* t_{tor.} \approx \frac{3 M_t}{1.6 * b^2} = \frac{3 * 62.7 * 10^6}{1.6 * 400^2} = 734.7 \text{ mm} \quad \boxed{t = 750 \text{ mm}}$$

Design the beam For bending. (400 * 750)



All Sections are R-Sec.



Sec ① $M_{U.L.} = 119.7 \text{ kN.m}$

$$700 = C_1 \sqrt{\frac{119.7 * 10^6}{25 * 400}} \rightarrow C_1 = 6.40 \rightarrow J = 0.826$$

$$A_s = \frac{119.7 * 10^6}{0.826 * 360 * 700} = 575.0 \text{ mm}^2$$

Check $A_{s_{min.}}$ $A_{s_{req.}} = 575.0 \text{ mm}^2$

$$\mu_{min.} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y} \right) b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 400 * 700 = 875 \text{ mm}^2$$

$\therefore \mu_{min.} b d > A_{s_{req.}} \xrightarrow{\text{Use}} A_{s_{min.}}$

$$A_{s_{min.}} = 0.225 * \frac{\sqrt{F_{cu}}}{F_y} b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 400 * 700 = 875$$

$$1.3 A_{s_{req.}} = 1.3 * 575.0 = 747.5$$

$$\text{st. 360/520} \quad \frac{0.15}{100} b d = \frac{0.15}{100} * 400 * 700 = 420 \text{ mm}^2$$

الأقل = 747.5
الأكبر = 747.5 mm²

Check if the Stirrups 5 #10 \ m + 5 #12 \ m are safe.

Sec. ①

$$q_u = \frac{Q}{bd} = \frac{129.8 * 10^3}{400 * 700} = 0.463 \text{ N/mm}^2$$

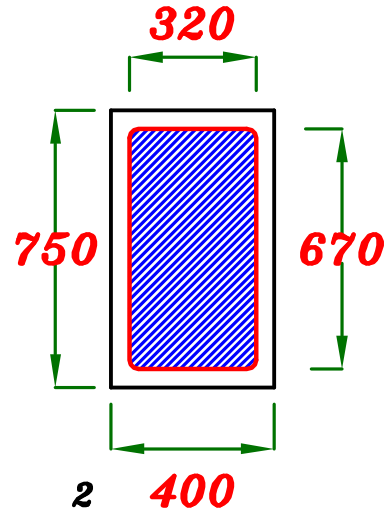
$$A_{oh} = 320 * 670 = 214400 \text{ mm}^2$$

$$A_o = 0.85 * A_{oh} = 0.85 * 214400 = 182240 \text{ mm}^2$$

$$P_h = 2 * 320 + 2 * 670 = 1980 \text{ mm}$$

$$t_e = \frac{A_{oh}}{P_h} = \frac{214400}{1980} = 108.28 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{62.7 * 10^6}{2 * 182240 * 108.28} = 1.59 \text{ N/mm}^2$$



$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2$$

$$q_{tmin} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245 \text{ N/mm}^2$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85 \text{ N/mm}^2$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.463^2 + 1.59^2} = 1.656 \text{ N/mm}^2 < q_{u_{max}} \therefore \text{o.k.}$$

$q_u < q_{cu}$, $q_{tu} > q_{tmin}$ \therefore The Stirrups will resist Torsion only.

$$A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\phi_s} \right)} = \frac{(62.7 * 10^6) * S_t}{(1.7)(214400)(360/1.15)}$$

* The given $\phi 12 \rightarrow A_{str} = 113 \text{ mm}^2 \therefore S_t = 1.819 * A_{str}$

$$\therefore S_t = 1.684 * A_{str} = 1.819 * 113 = 205.54 \text{ mm}$$

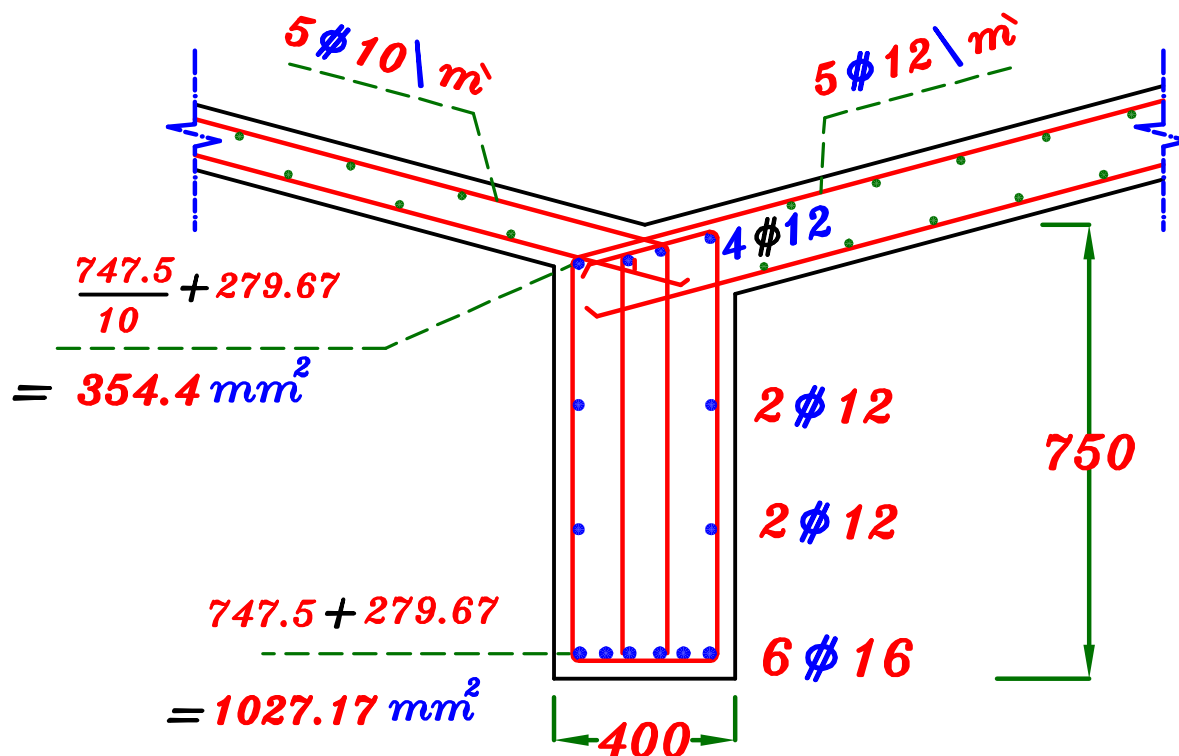
$$\therefore \text{No. of stirrups required} \setminus m \setminus = \frac{1000}{S} = \frac{1000}{205.54} = 4.86 < 5.0$$

\therefore the stirrups $5\phi 12 \setminus m \setminus$ is enough to resist the Torsion.

* Longitudinal Bars. $S_t = \frac{1000}{5} = 200 \text{ mm}$

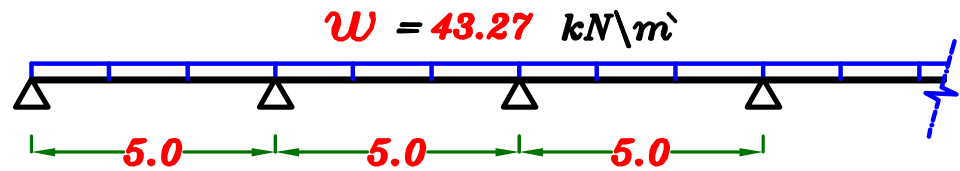
$$A_{sl} = \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = \frac{(113 * 1980)}{200} \left(\frac{360}{360} \right) = 1118.7 \text{ mm}^2$$

$$\therefore \frac{A_{sl}}{4} = \frac{1118.7}{4} = 279.67 \text{ mm}^2$$

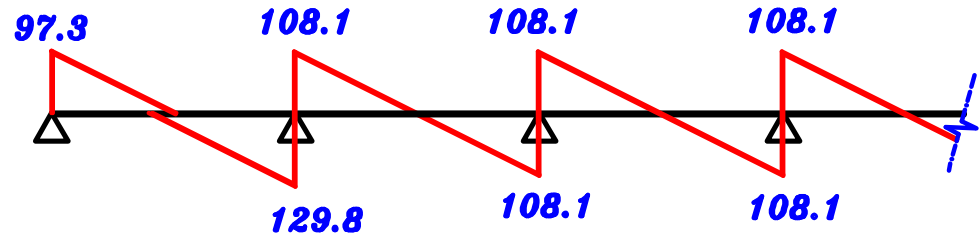


Normal Force & Bending Moment on the column.

Load



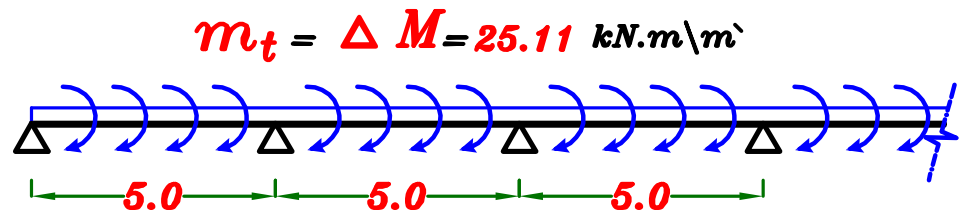
S.F.D.
For the beam.



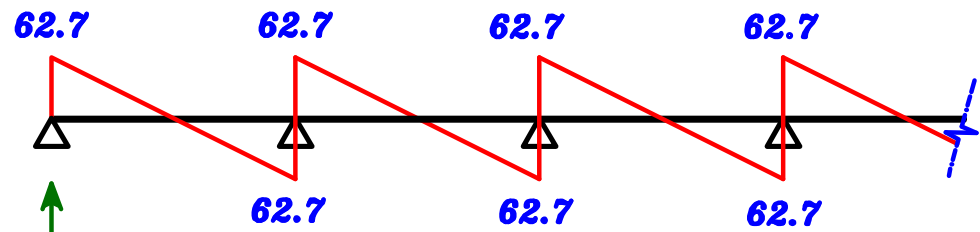
Reaction of S.F.D.
=
Load on the column.



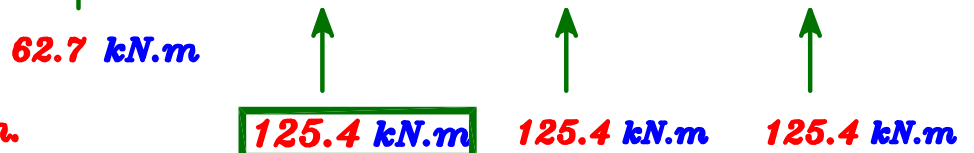
Torsion



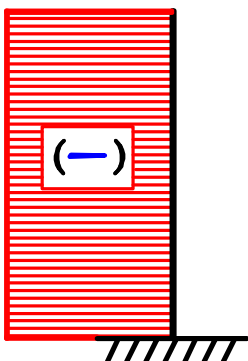
T.M.D.
For the beam.



Reaction of T.M.D.
=
Bending on the column.

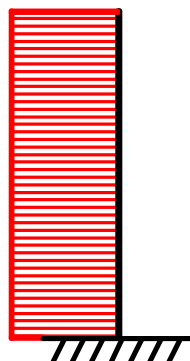


237.9 kN

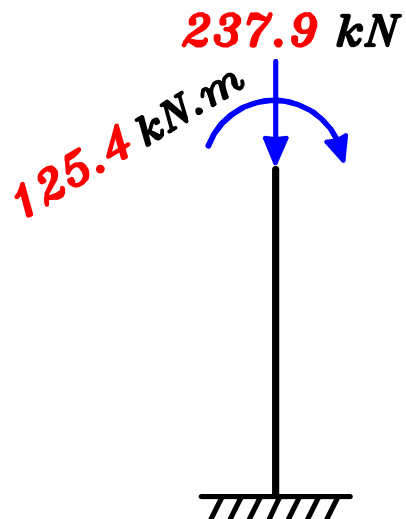


N.F.D.

125.4 kN.m



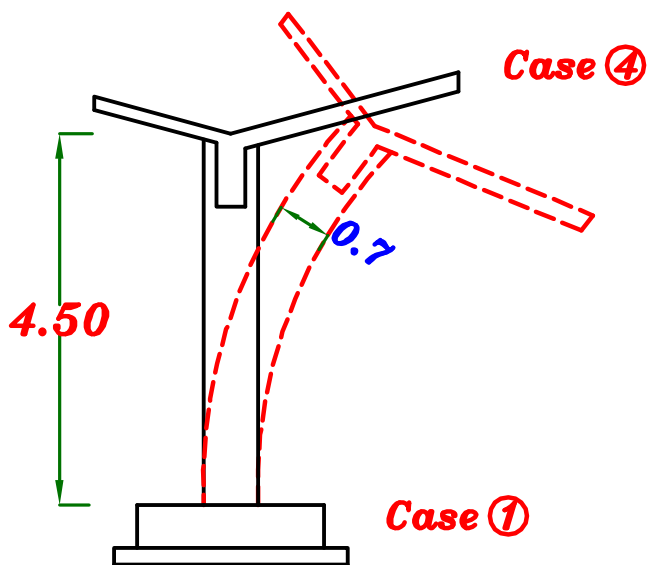
B.M.D.



Take the Column (400*700)

Check Buckling.

① In plane.

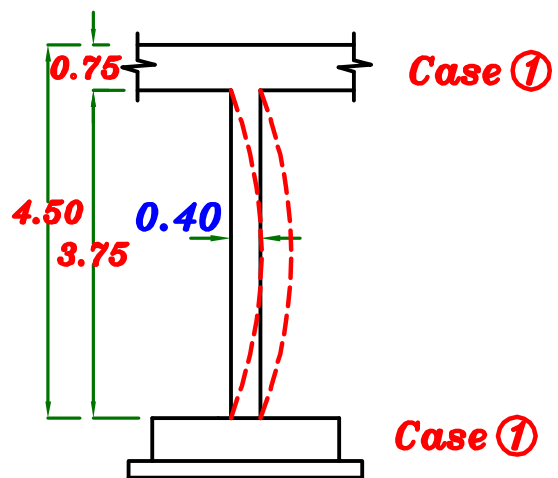


Upper Condition Case ④ } $k = 2.2$
Lower Condition Case ① }

$$H_o = 4.50 \text{ m}$$

$$\lambda_{b_{in}} = \frac{2.2 * 4.5}{0.7} = 14.14 > 10$$

② Out of plane.



Upper Condition Case ① } $k = 1.2$
Lower Condition Case ① }

$$H_o = 3.75 \text{ m}$$

$$\lambda_{b_{out}} = \frac{1.2 * 3.75}{0.40} = 11.2 > 10$$

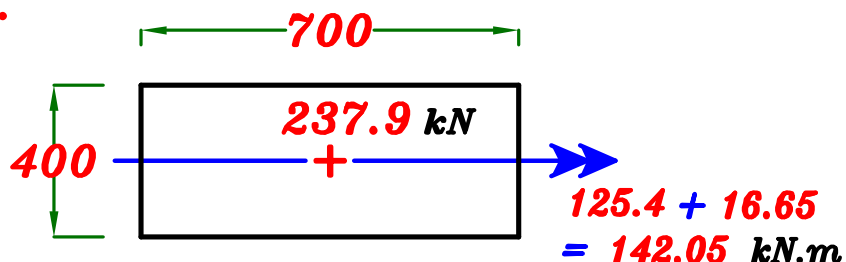
Take the bigger value of $\lambda_b = 14.1$ (In plane)

$$\delta = \frac{(\lambda_b)^2 * t}{2000} = \frac{14.14^2 * 0.7}{2000} = 0.07 \text{ m}$$

$$M_{add.} = P * \delta = 237.9 * 0.07 = 16.65 \text{ kN.m}$$

$$\therefore M_{des.} = M_{ext.} + M_{add.}$$

$$\begin{aligned} \therefore M_{des.} &= 125.4 + 16.65 \\ &= 142.05 \text{ kN.m} \end{aligned}$$



Design the Sec.

$$e = \frac{M}{P} = \frac{142.05}{237.9} = 0.597 \text{ m} \quad \therefore \frac{e}{t} = \frac{0.597}{0.7} = 0.85 > 0.5 \xrightarrow{\text{use}} e_s$$

$$e_s = e + \frac{t}{2} - c = 0.597 + \frac{0.70}{2} - 0.05 = 0.897 \text{ m}$$

$$M_s = P * e_s = 237.9 * 0.897 = 213.4 \text{ kN.m}$$

$$\therefore 650 = C_1 \sqrt{\frac{213.4 * 10^6}{25 * 400}} \longrightarrow C_1 = 4.45 \longrightarrow J = 0.817$$

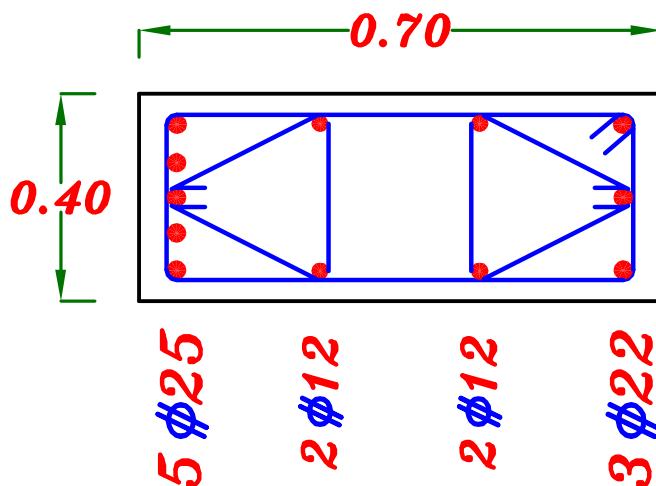
$$\begin{aligned} \therefore A_s &= \frac{M_s}{J F_y d} - \frac{P_{U.L.}}{(F_y \setminus \phi_s)} \\ &= \frac{213.4 * 10^6}{0.817 * 360 * 650} - \frac{237.9 * 10^3}{(360 \setminus 1.15)} = 356.27 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} A_{s_{min}} &= \frac{0.25 + 0.052 \lambda_{max}}{100} * b * t \\ &= \frac{0.25 + 0.052 (14.14)}{100} * 400 * 700 = 2758.8 \text{ mm}^2 > A_s \end{aligned}$$

$$\therefore \text{Take } A_s = A_{s_{min}} = 2239.3 \text{ mm}^2 \quad (5 \phi 25)$$

$$\text{Stirrup Hangers} = 0.4 A_s = 0.4 * 2239.3 = 895.7 \text{ mm}^2$$

$$(3 \phi 22)$$



Example.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

Beam (300 * 750)

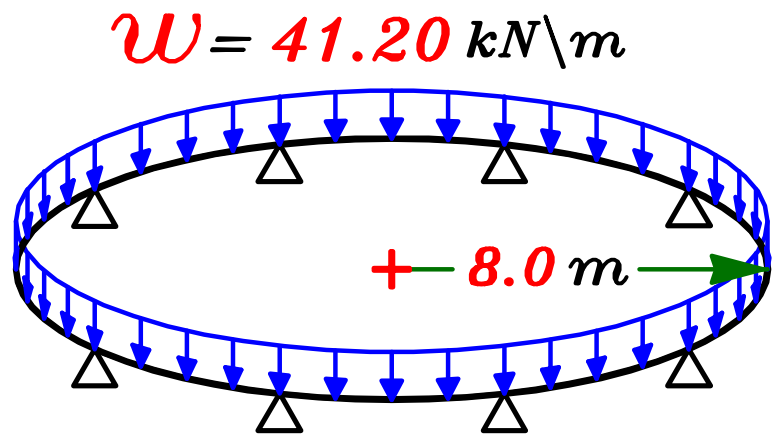
No. of supports = 8.0

$$W_{\text{working}} = 41.20 \text{ kN/m}$$

Radius of the beam = 8.0 m

Req.

- 1- Calculate the straining action on the beam.
- 2- Design the beam.
- 3- Draw details of RFT. in elevation and cross section of the beam.



From Tables

No. of supports	Load on each support	Max. Shearing Force	Max. Bending Moment		Max. Torsional Moment	Central angle
			at C.L. of Span	Over C.L. of Column		
n	R	$Q_{\text{max.}}$	$M +ve$	$M -ve$	$M_{t \text{ max.}}$	θ
4	$P/4$	$P/8$	$0.0176 Pr$	$-0.0322 Pr$	$0.0053 Pr$	$19^\circ 21'$
6	$P/6$	$P/12$	$0.0075 Pr$	$-0.0148 Pr$	$0.0015 Pr$	$12^\circ 44'$
8	$P/8$	$P/16$	$0.0042 Pr$	$-0.0083 Pr$	$0.0006 Pr$	$9^\circ 33'$
10	$P/10$	$P/20$	$0.0032 Pr$	$-0.0052 Pr$	$0.0004 Pr$	$7^\circ 36'$
12	$P/12$	$P/24$	$0.0019 Pr$	$-0.0037 Pr$	$0.0002 Pr$	$6^\circ 21'$

$$r = 8.0$$

$$P = w * 2 \pi r = 41.20 * 2 \pi * 8.0 = 2070.93 \text{ kN}$$

$$\text{max. } M_{+ve} = 0.0042 P r = 0.0042 * 2070.93 * 8.0 = 69.58 \text{ kN.m}$$

$$\text{max. } M_{-ve} = 0.0083 P r = 0.0083 * 2070.93 * 8.0 = 137.51 \text{ kN.m}$$

$$\text{max. } M_t = 0.0006 P r = 0.0006 * 2070.93 * 8.0 = 9.94 \text{ kN.m}$$

$$Q_{\text{max.}} = \frac{P}{16} = \frac{2070.93}{16} = 129.43 \text{ kN}$$

$$\text{Central angle } \theta = 9^\circ 33' = 9.55^\circ$$

$$X = r * \theta * \frac{\pi}{180} = 8.0 * 9.55 * \frac{\pi}{180} = 1.33 \text{ m}$$

$$Q_{\text{cor.}} = Q_{\text{max}} - w * X = 129.43 - 41.20 * 1.33 = 74.63 \text{ kN}$$

$$\text{Design beam B1} \quad b = 300 \text{ mm} , t = 750 \text{ mm}$$

$$\text{Sec. of max - } V_e .$$

$$M = 137.51 * 1.5 = 206.26 \text{ kN.m.}$$

$$\therefore d = C_1 \sqrt{\frac{M_{U.L.}}{F_{cu} b}} \therefore 700 = C_1 \sqrt{\frac{206.26 * 10^6}{25 * 300}} \rightarrow C_1 = 4.22 \rightarrow J = 0.810$$

$$\therefore A_s = \frac{M_{U.L.}}{J F_y d} = \frac{206.26 * 10^6}{0.810 * 360 * 700} = 1010.5 \text{ mm}^2$$

$$\text{Check } A_{s_{\text{min.}}} \quad A_{s_{\text{req.}}} = 1010.5 \text{ mm}^2$$

$$\mu_{\text{min.}} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y} \right) b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 300 * 700 = 656.25 \text{ mm}^2$$

$$\therefore A_{s_{\text{req.}}} > \mu_{\text{min.}} b d \therefore \text{Take } A_s = A_{s_{\text{req.}}} = 1010.5 \text{ mm}^2$$

Sec. of $\max + Ve$.

$$M = 69.58 * 1.5 = 104.37 \text{ kN.m.}$$

$$\therefore d = C_1 \sqrt{\frac{M_{U.L.}}{F_{cu} b}} \quad \therefore 700 = C_1 \sqrt{\frac{104.37 * 10^6}{25 * 300}} \rightarrow C_1 = 5.93 \rightarrow J = 0.826$$

$$\therefore A_s = \frac{M_{U.L.}}{J F_y d} = \frac{104.37 * 10^6}{0.826 * 360 * 700} = 501.4 \text{ mm}^2$$

Check $A_{s_{min.}}$ $A_{s_{req.}} = 501.4 \text{ mm}^2$

$$\mu_{min.} b d = \left(0.225 * \frac{\sqrt{F_{cu}}}{F_y} \right) b d = \left(0.225 * \frac{\sqrt{25}}{360} \right) 300 * 700 = 656.25 \text{ mm}^2$$

$$\therefore \mu_{min.} b d > A_{s_{req.}} \xrightarrow{\text{Use}} A_{s_{min.}}$$

$$\left. \begin{aligned} A_{s_{min.}} &= \left(0.225 * \frac{\sqrt{25}}{360} \right) 300 * 700 = 656.25 \text{ mm}^2 \\ 1.3 A_{s_{req.}} &= 1.3 * 501.4 = 651.8 \text{ mm}^2 \end{aligned} \right\} \text{الأقل} = 651.8 \text{ mm}^2$$

Design due to Shear & Torsion.

$$q_u = \frac{Q}{b d} = \frac{1.5 * 74.63 * 10^3}{300 * 700} = 0.533 \text{ N/mm}^2$$

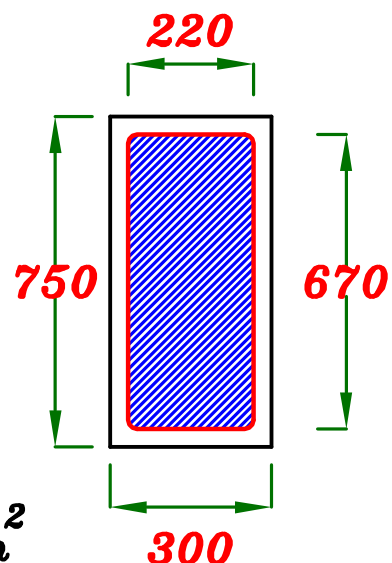
$$A_{oh} = 220 * 670 = 147400 \text{ mm}^2$$

$$A_o = 0.85 * A_{oh} = 0.85 * 147400 = 125290 \text{ mm}^2$$

$$P_h = 2 * 220 + 2 * 670 = 1780 \text{ mm}$$

$$t_e = \frac{A_{oh}}{P_h} = \frac{147400}{1780} = 82.81 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{1.5 * 9.94 * 10^6}{2 * 125290 * 82.81} = 0.72 \text{ N/mm}^2$$



$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2$$

$$q_{tmin} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245 \text{ N/mm}^2$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85 \text{ N/mm}^2$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.533^2 + 0.72^2} = 0.896 \text{ N/mm}^2 < q_{u_{max}} \therefore \text{o.k.}$$

$$q_u < q_{cu}, q_{tu} > q_{tmin} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\delta_s}\right)} \therefore A_{str} = \frac{(1.5 \cdot 9.94 \cdot 10^6) \cdot S_t}{(1.7)(147400)(240/1.15)}$$

$$\therefore S_t = 3.507 \cdot A_{str}$$

$$* \text{ Take } \phi 8 \rightarrow A_{str} = 50.3 \text{ mm}^2$$

$$\therefore S_t = 3.507 \cdot A_{str} = 3.507 \cdot 50.3 = 176.4 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

$$\therefore \text{No. of stirrups/m} = \frac{1000}{S} = \frac{1000}{176.4} = 5.66 = 6.0$$

$$\therefore \text{Use Closed Stirrups } 6 \phi 8 \setminus \text{m} \quad 2 \text{ branches.}$$

* Longitudinal Bars.

$$S_t = \frac{1000}{6} = 166.66 \text{ mm}$$

$$A_{sl} = \frac{A_{str} * P_h}{S_t} \left(\frac{F_{y \text{ str.}}}{F_{y \text{ L.b.}}} \right) = \frac{(50.3 * 1780)}{166.66} \left(\frac{240}{360} \right) = 358.15 \text{ mm}^2$$

$$\therefore \frac{A_{sl}}{4} = \frac{358.15}{4} = 89.53 \text{ mm}^2$$

$$A_{s - ve} = A_s + \frac{A_{sl}}{4} = 1010.5 + 89.53 = 1100.03 \text{ mm}^2$$

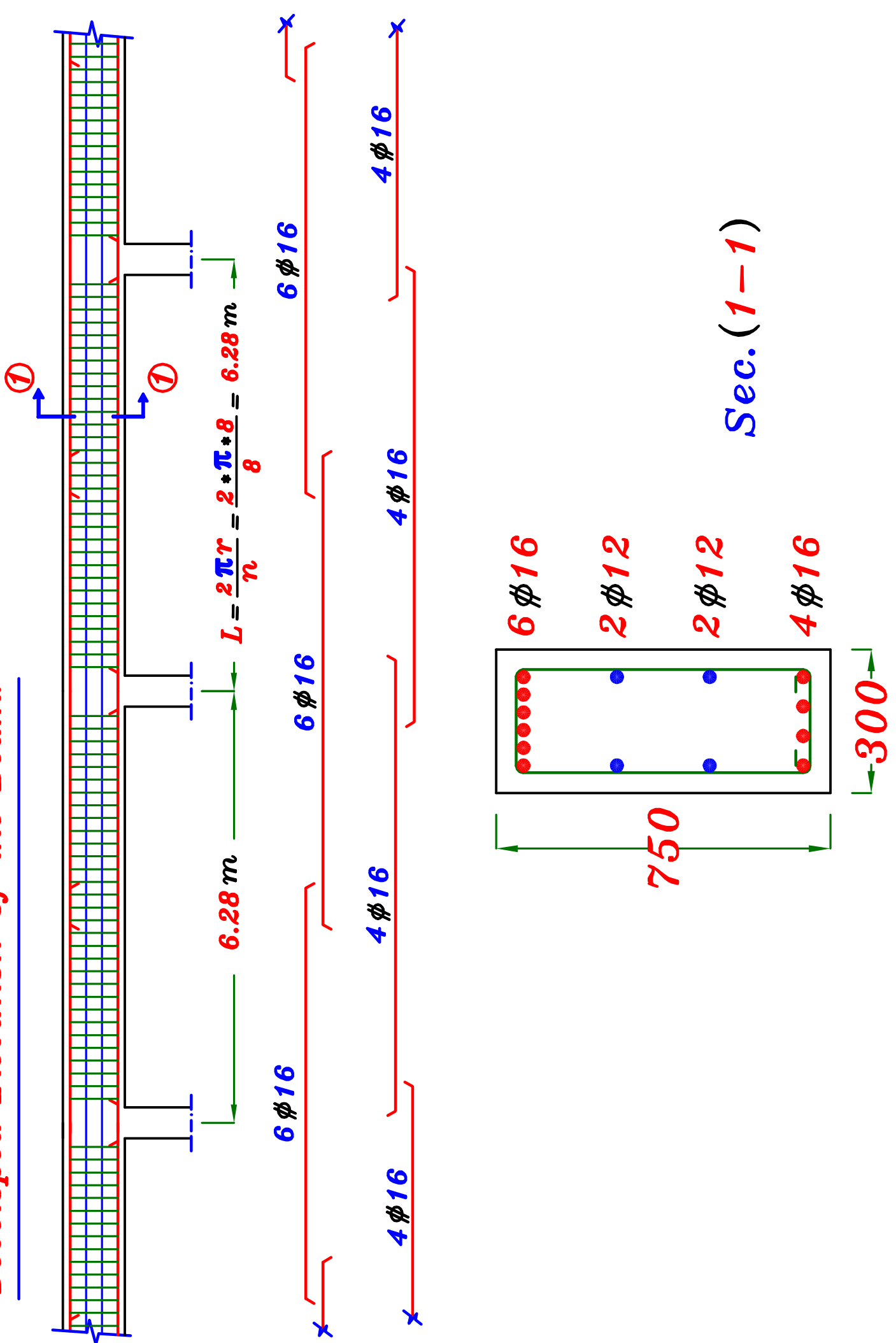
$$6 \phi 16$$

$$\therefore n = \frac{b - 25}{\phi + 25} = \frac{300 - 25}{16 + 25} = 6.70 = 6.0$$

$$A_{s + ve} = A_s + \frac{A_{sl}}{4} = 651.8 + 89.53 = 741.33 \text{ mm}^2$$

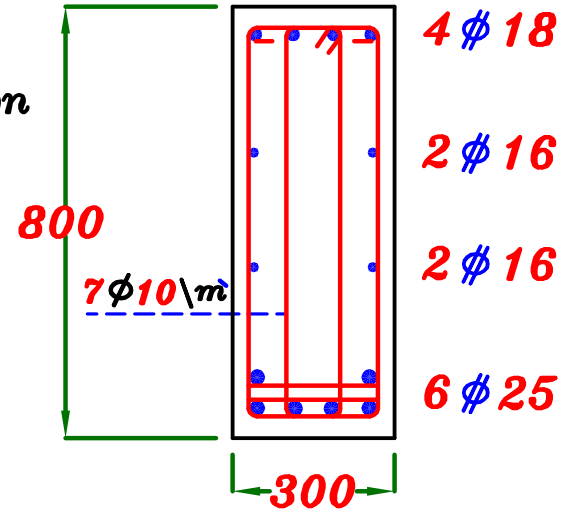
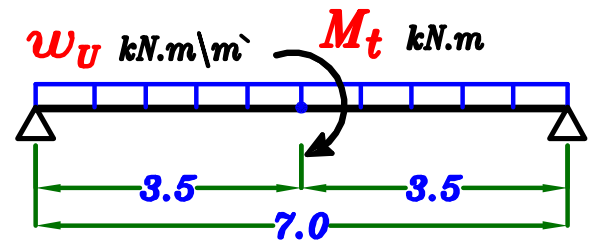
$$4 \phi 16$$

Developed Elevation of the Beam.



Example.

The Following Fig. shows a simple girder subjected to uniform distributed Factored load (w_U) including own weight and local torque moment (M_t) at mid span. It is required to calculate the max. torque moment (M_t), For the girder and its section given in the Figure.



Data.

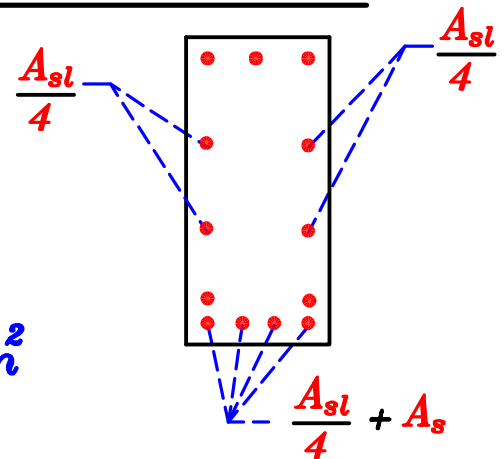
$$F_{cu} = 30 \text{ N/mm}^2$$

$$F_y = 400 \text{ N/mm}^2$$

Solution.

$$\frac{A_{sl}}{4} = 2 \# 16$$

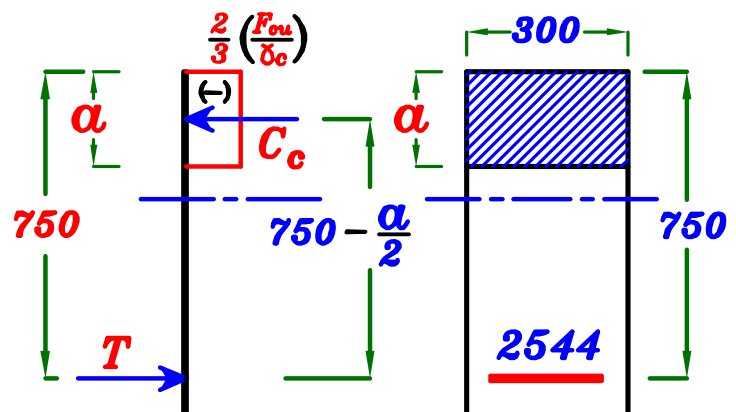
$$\begin{aligned} A_s (\text{From moment}) &= 6 \# 25 - 2 \# 16 \\ &= 2946 - 402 = 2544 \text{ mm}^2 \end{aligned}$$



Use First Principles to get $M_{U.L.}$

$$\begin{aligned} \frac{2}{3} \frac{F_{cu}}{\gamma_c} * a * b &= A_s * \frac{F_y}{\gamma_s} \\ \frac{2}{3} \left(\frac{30}{1.5} \right) (a) (300) &= (2544) \left(\frac{400}{1.15} \right) \end{aligned}$$

$$a = 221.2 \text{ mm} > 0.1 d$$



$$\begin{aligned} M_{U.L.} &= \frac{2}{3} \frac{F_{cu}}{\gamma_c} a b \left(d - \frac{a}{2} \right) = \frac{2}{3} \left(\frac{30}{1.5} \right) (221.2) (300) \left(750 - \frac{221.2}{2} \right) \\ &= 565741120 \text{ N.mm} = 565.7 \text{ kN.m} \end{aligned}$$

$$\therefore M_{U.L.} = 565.7 = \frac{w_U (7.0)^2}{8}$$

$$\therefore w_U = 92.35 \text{ kN/m}$$

$$Q_{max} = \frac{w * L}{2} = 323.2 \text{ kN}$$

$$q_u = \frac{Q}{bd} = \frac{323.2 * 10^3}{300 * 750} = 1.436 \text{ N/mm}^2$$

$$\therefore q_{cu} = (0.24) \sqrt{\frac{30}{1.5}} = 1.07 \text{ N/mm}^2$$

$$S_s = S_t = \frac{1000}{7.0} = 142.85 \text{ mm}$$

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y / \delta_s)}{b S_s} \quad \therefore 1.436 - \frac{1.07}{2} = \frac{4 * A_s (240 / 1.15)}{(300) (142.85)}$$

$$\therefore A_s = 46.25 \text{ mm}^2$$

$$\therefore A_{s_{outer}} = A_{str} + A_s = A_{str} + 46.25 = 78.5 \text{ mm}^2 \quad A_{str} = 32.25 \text{ mm}^2$$

$$x_1 = 220 \text{ mm}, y_1 = 720 \text{ mm}, A_{oh} = 220 * 720 = 158400 \text{ mm}^2$$

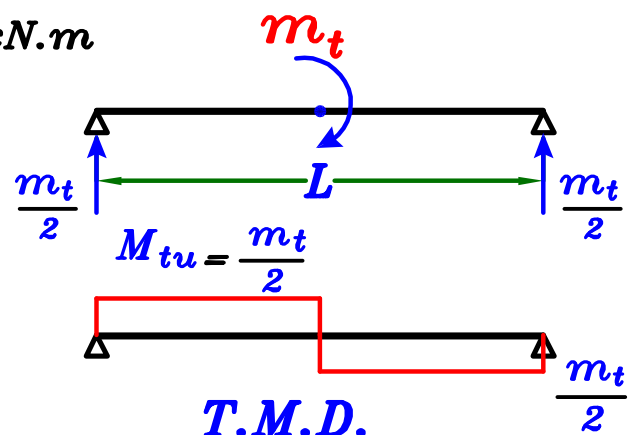
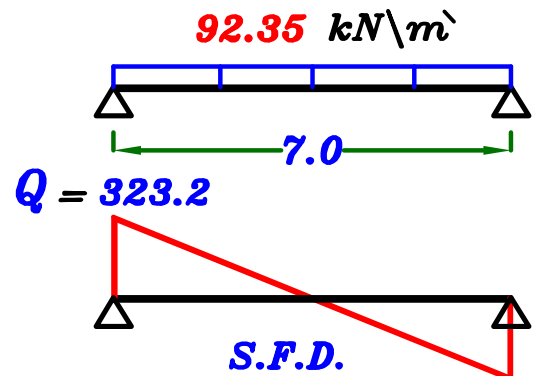
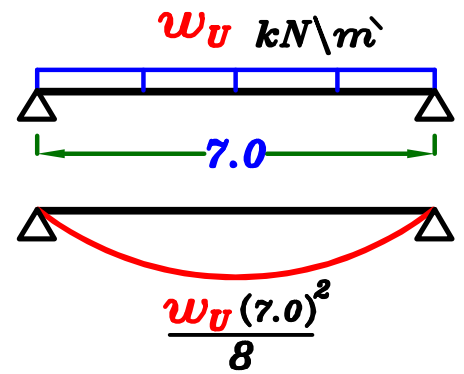
$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} \left(\frac{F_y}{\delta_s} \right)} \quad \therefore 32.25 = \frac{M_{tu} * (142.85)}{(1.7) (158400) (240 / 1.15)}$$

$$\therefore M_{tu} = 12687234.7 \text{ N.mm} = 12.68 \text{ kN.m}$$

$$\therefore M_{tu} = \frac{m_t}{2}$$

$$\therefore m_t = 12.68 * 2 = 25.36 \text{ kN.m}$$

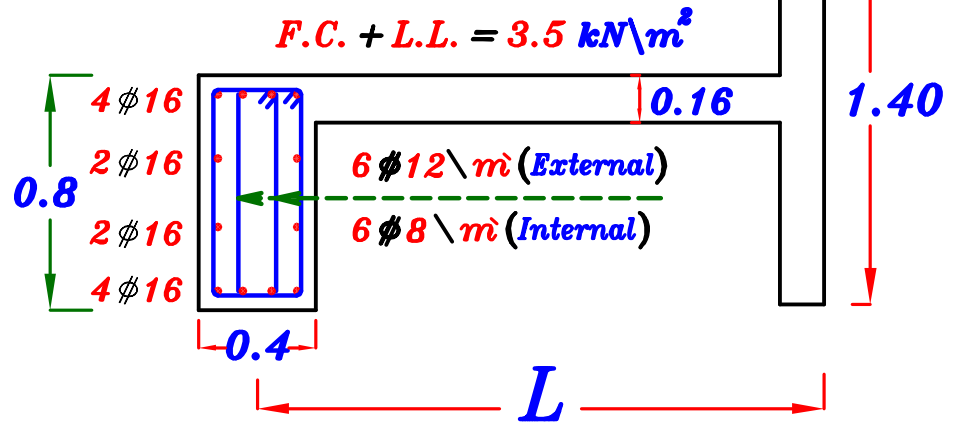
$$\therefore M_t = 25.36 \text{ kN.m}$$



Example.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$



The Figure shows the cross-section dimensions and RFT. of continuous beam supported on columns spaced **6.0 m**. The beam has a cantilever solid slab of **0.16 m** thickness. It is required to calculate the safe length of the cantilever slab to satisfy the RFT. & loads shown in Fig.

$$w_s = 1.5 (0.16 * 25 + 3.50) = 11.25 \text{ kN/m}^2$$

$$\text{parapet weight} = 1.4 (0.15 * 1.4 * 1.0) * 25 = 7.35 \text{ kN/m}$$

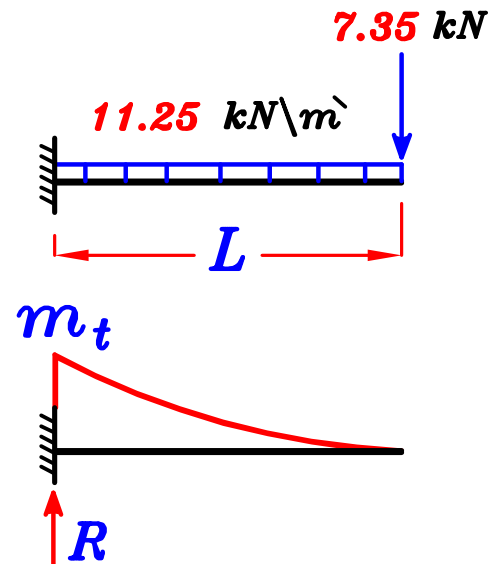
$$\text{o.w. (beam)} = 1.4 (0.4 * 0.8 * 1.0) * 25 = 11.2 \text{ kN/m}$$

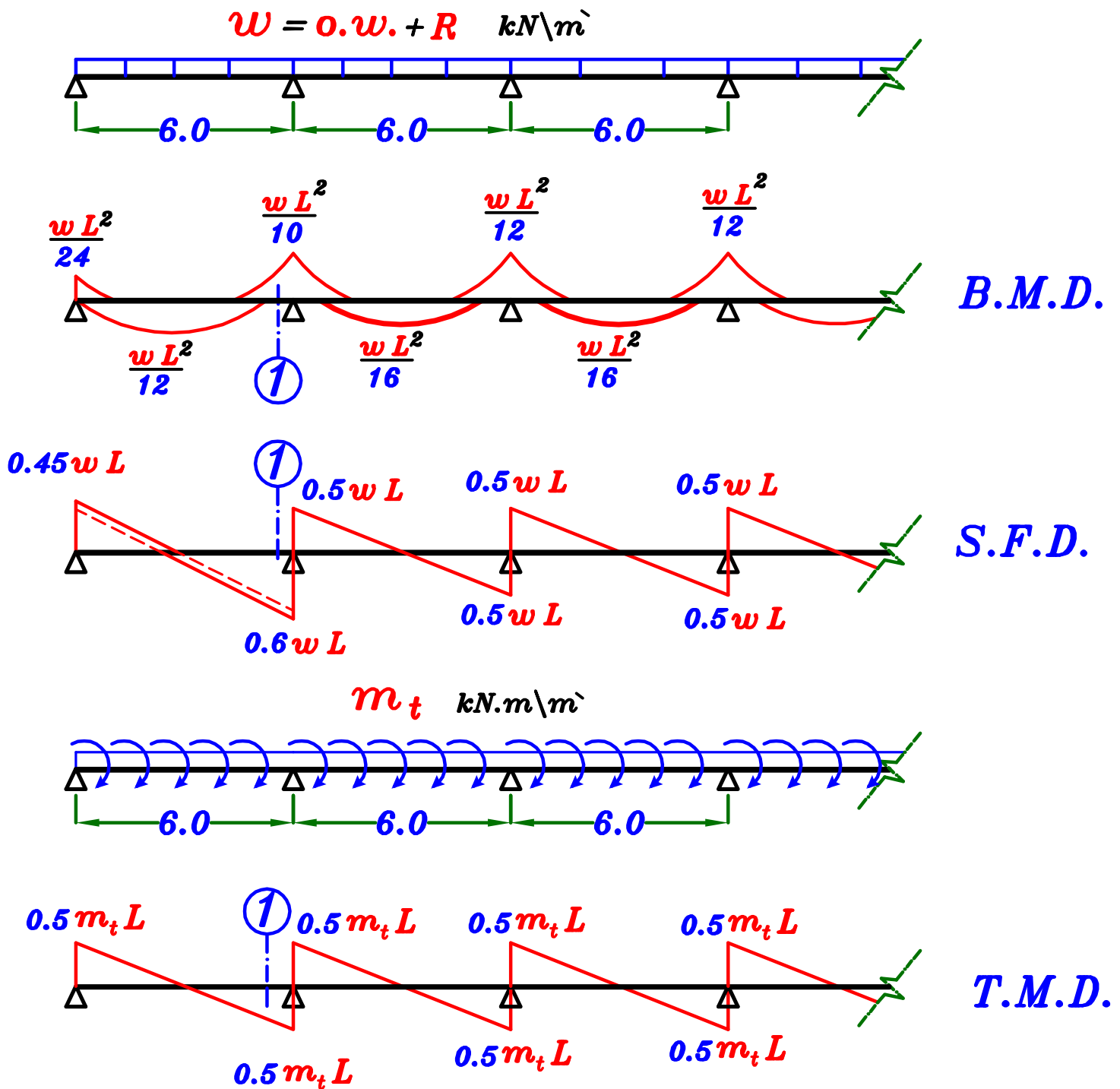
Strip in the slab.

$$R = 11.25 L + 7.35 \text{ kN}$$

$$\begin{aligned} m_t &= 7.35 L + 11.25 \frac{L^2}{2} \text{ kN.m} \\ &= 7.35 L + 5.625 L^2 \end{aligned}$$

$$\begin{aligned} w &= \text{o.w.} + R = 11.2 + (11.25 L + 7.35) \text{ kN/m} \\ &= 18.55 + 11.25 L \end{aligned}$$





For Sec. ①

$$M_{U.L.} = \frac{wL^2}{10} = \frac{(18.55 + 11.25L) * 6.0^2}{10} \quad \therefore \boxed{M_{U.L.} = 66.78 + 40.5L} \quad kN.m$$

$$Q = 0.6wL = 0.6 (18.55 + 11.25L) * 6.0 \quad \therefore \boxed{Q_{U.L.} = 66.78 + 40.5L} \quad kN$$

$$M_t = 0.5 m_t L = 0.5 (7.35L + 5.625L^2) * 6.0$$

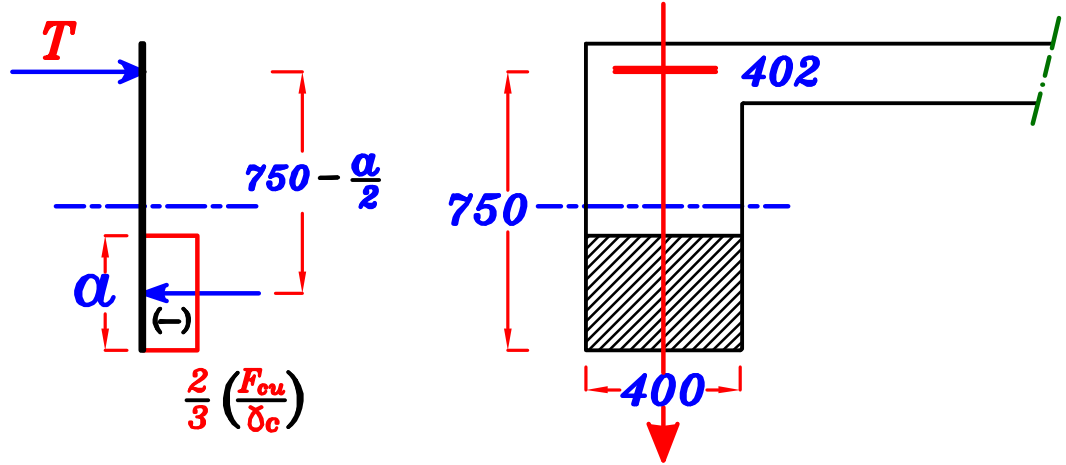
$$\therefore \boxed{M_t = 22.05L + 16.87L^2} \quad kN.m$$

For Bending Moment.

$$\frac{A_{sl}}{4} = 2 \phi 16$$

$$A_s (\text{From moment}) = 4 \phi 16 - 2 \phi 16 = 804 - 402 = 402 \text{ mm}^2$$

Neglect A_s'



Use First Principles to get $M_{U.L.}$

$$\frac{2}{3} \frac{F_{cu}}{\delta_c} * \alpha * b = A_s * \frac{F_y}{\delta_s}$$

$$\frac{2}{3} \left(\frac{25}{1.5} \right) (\alpha) (400) = (402) \left(\frac{360}{1.15} \right) \longrightarrow \alpha = 28.3 \text{ mm} < 0.1 d$$

$$M_{U.L.} = 0.826 A_s F_y d = 0.826 (402) (360) (750) \\ = 89654040 \text{ N.mm} = 89.65 \text{ kN.m}$$

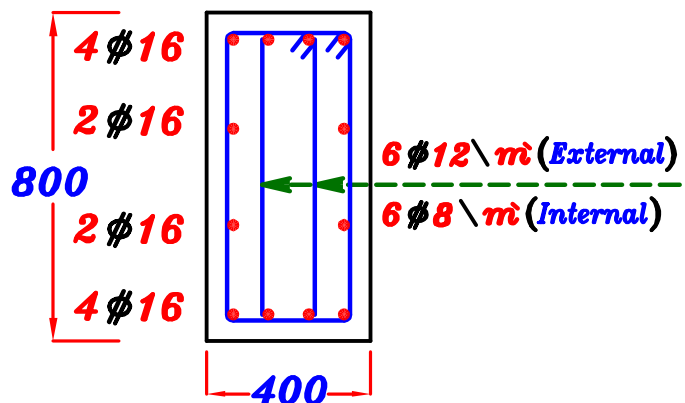
$$\therefore M_{U.L.} = 89.65 \text{ kN.m}$$

$$n = 4.0$$

$$A_{s_{inner}} = \phi 8 = A_s = 50.3 \text{ mm}^2$$

$$A_{s_{outer}} = \phi 12 = A_{str} + A_s = 113 \text{ mm}^2$$

$$\therefore A_{str} = 113 - 50.3 = 62.7 \text{ mm}^2$$



For Shear

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98 \text{ N/mm}^2, \quad S_s = S_t = \frac{1000}{6.0} = 166.67 \text{ mm}$$

$$\therefore q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \gamma_s)}{b S_s}$$

$$\therefore q_u - \frac{0.98}{2} = \frac{4 (50.3) (360 \gamma_s)}{(400) (166.67)} \longrightarrow q_u = 1.434 \text{ N/mm}^2$$

$$\therefore q_u = \frac{Q}{b d} \quad \therefore 1.434 = \frac{Q}{(400) (750)} \longrightarrow Q = 430200 \text{ N}$$

$$\therefore \boxed{Q_{U.L.} = 430.2 \text{ kN}}$$

For Torsional Moment.

$$A_{s_{inner}} = \phi 8 = A_s = 50.3 \text{ mm}^2$$

$$A_{s_{outer}} = \phi 12 = A_{str} + A_s = 113 \text{ mm}^2$$

$$\therefore A_{str} = 113 - 50.3 = 62.7 \text{ mm}^2$$

$$x_1 = 320 \text{ mm}, \quad y_1 = 720 \text{ mm}$$

$$A_{oh} = 320 * 720 = 230400 \text{ mm}^2$$

$$S_s = S_t = \frac{1000}{6.0} = 166.67 \text{ mm}$$

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) A_{oh} (F_y \setminus \phi_s)}$$

$$\therefore 62.7 = \frac{M_{tu} \cdot (166.67)}{(1.7)(230400) (360/1.15)} \longrightarrow M_{tu} = 46126039 \text{ N.mm}$$

$$\therefore \boxed{M_t = 46.12 \text{ kN.m}}$$

To get the safe length of the cantilever get the least of L_1, L_2, L_2

$$- M_{U.L.} = 89.65 \text{ kN.m} = 66.78 + 40.5 L_1 \longrightarrow L_1 = 0.564 \text{ m}$$

$$- Q_{U.L.} = 430.2 \text{ kN} = 66.78 + 40.5 L_2 \longrightarrow L_2 = 8.97 \text{ m}$$

$$- M_t = 46.12 \text{ kN.m} = 22.05 L_3 + 16.87 L_3^2 \longrightarrow L_3 = 1.12 \text{ m}$$

$$\therefore \boxed{L = 0.564 \text{ m}}$$

Example.

For The given Figure of structural plan.

It is required to :

Draw B.M.D. , S.F.D. & T.M.D. For the main Girder (G)

Data.

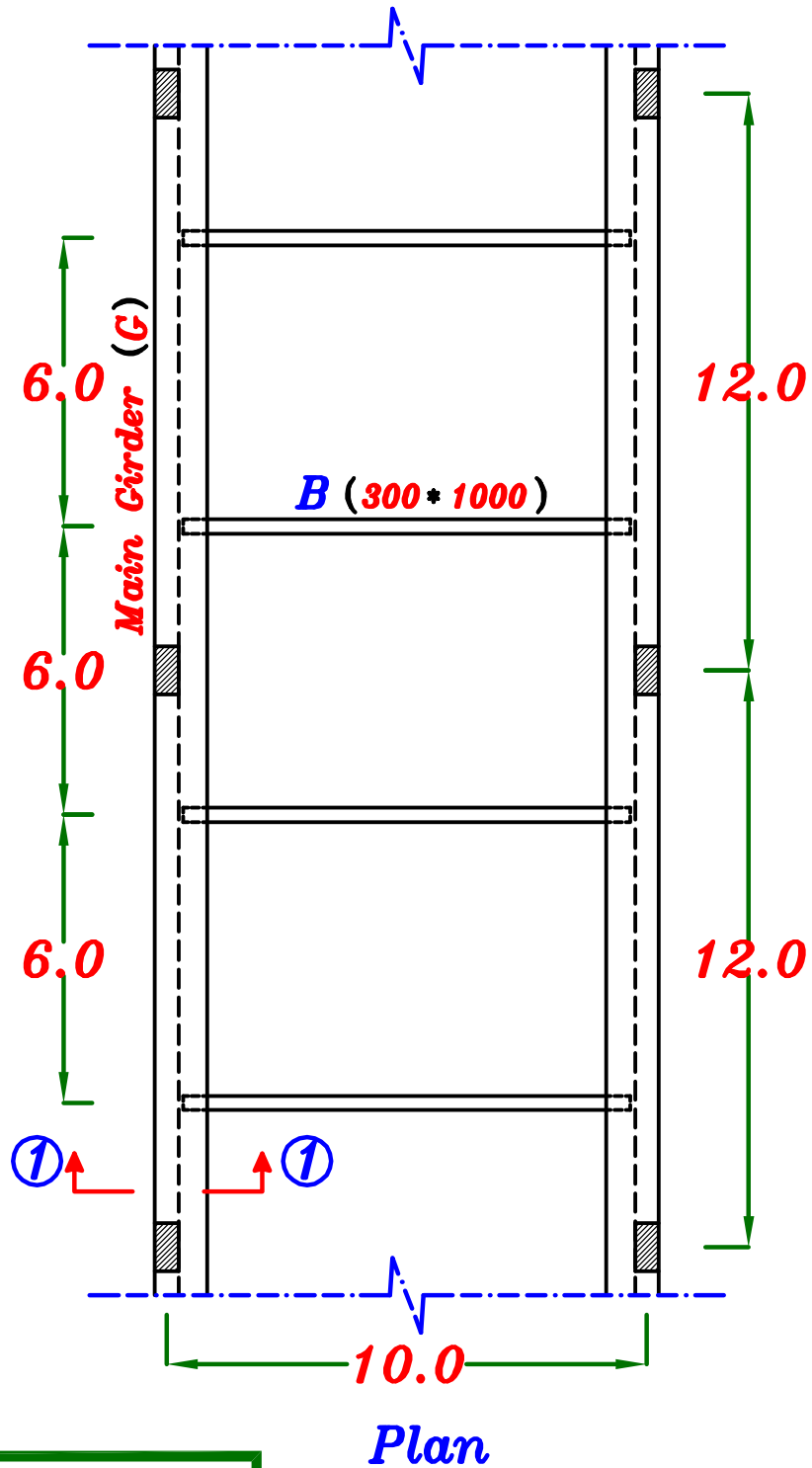
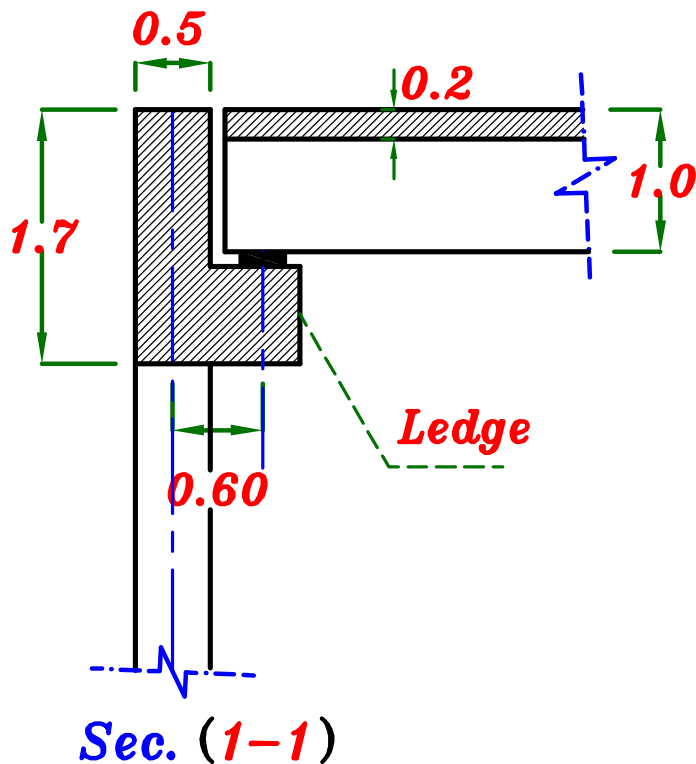
$$F_{cu} = 40 \text{ N/mm}^2$$

$$F_y = 400 \text{ N/mm}^2$$

$$F.C. = 3.0 \text{ kN/m}^2$$

$$L.L. = 5.0 \text{ kN/m}^2$$

$$t_s = 200 \text{ mm}$$



Neglect effect of Ledge إهمل تأثير النتوء
Design of Ledge is not required

Solution.

$$w_s = 1.4 (t_s \delta_c + F.C.) + 1.6 (L.L.)$$

$$w_s = 1.4 (0.20 * 25 + 3.0) + 1.6 (5.0) = 19.2 \text{ kN/m}^2$$

$$O.W. = 1.4 (b * t * \delta_c) = 1.4 (0.3 * 1.0 * 25) = 10.5 \text{ kN/m}$$

(Beam)

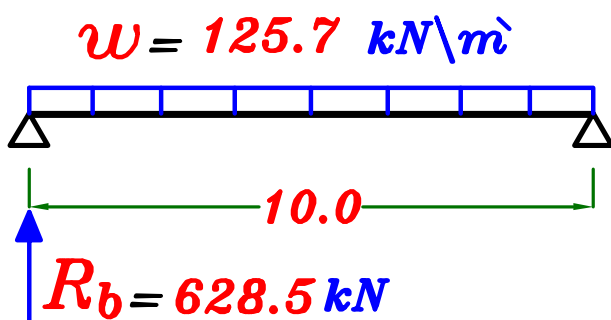
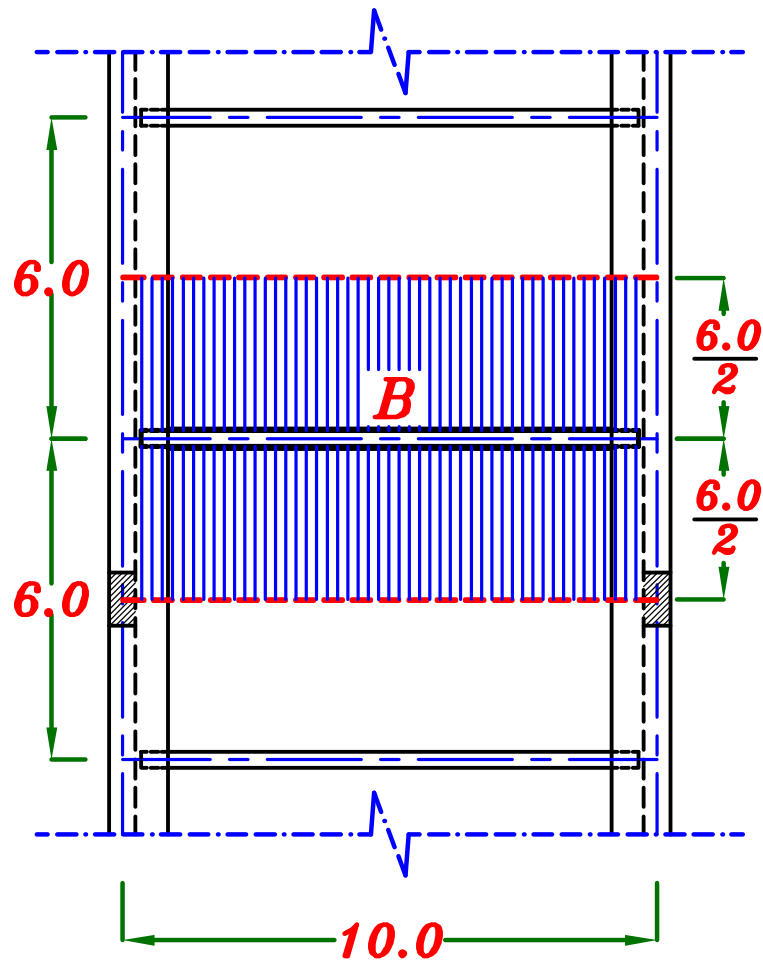
$$O.W. = 1.4 (b * t * \delta_c) = 1.4 (0.5 * 1.7 * 25) = 29.75 \text{ kN/m}$$

(Girder)

Neglect effect of Ledge إهمل تأثير النتوء

Beam.

$$\begin{aligned} W_{(Beam)} &= O.W. + 2 w_s \frac{L_s}{2} \\ &= 10.5 + 2 (19.2) \frac{6.0}{2} \\ &= 125.7 \text{ kN/m} \end{aligned}$$

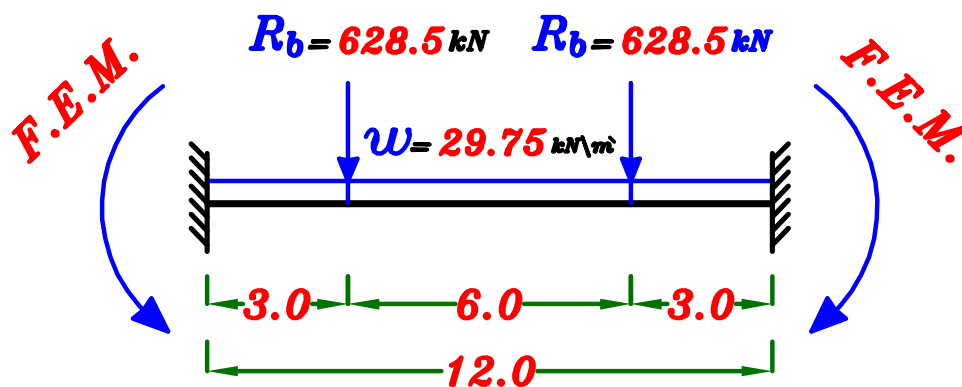
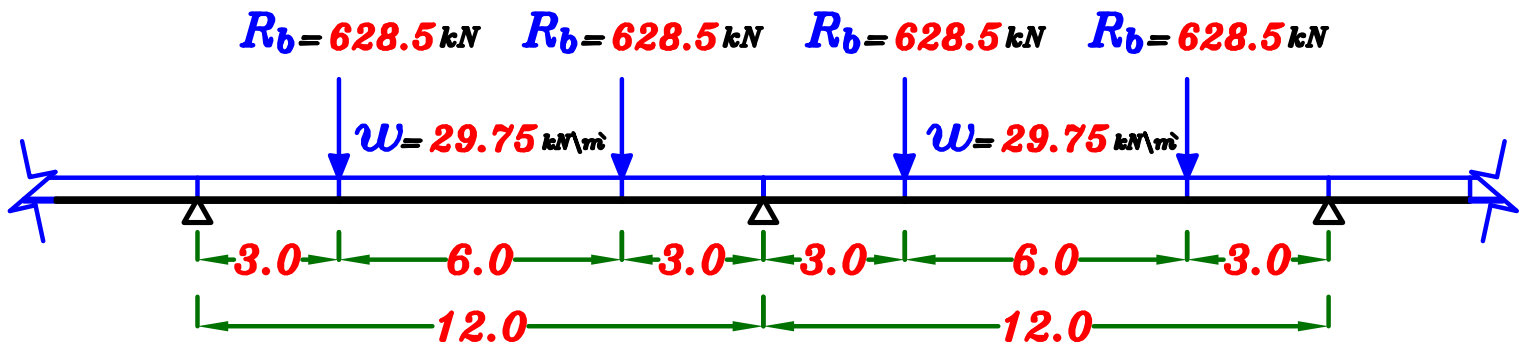


$$R_b = 628.5 \text{ kN}$$

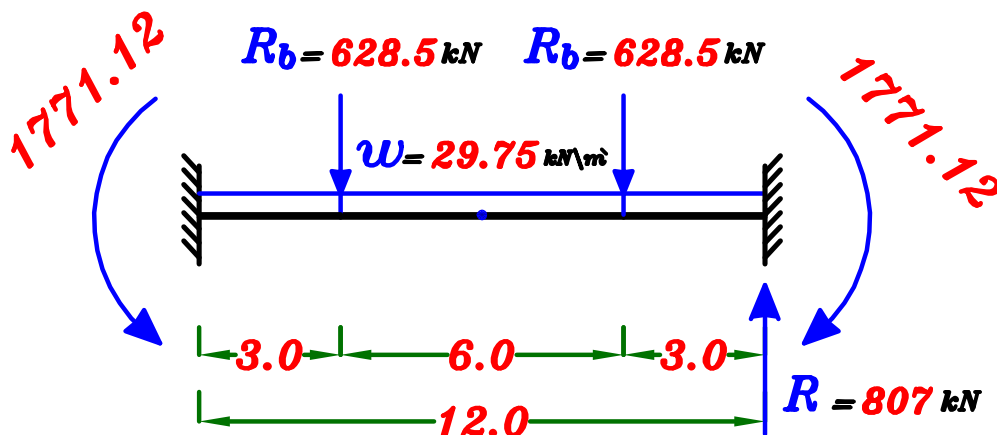
Main Girder.

$$W(\text{Girder}) = O.W. = 29.75 \text{ kN/m}$$

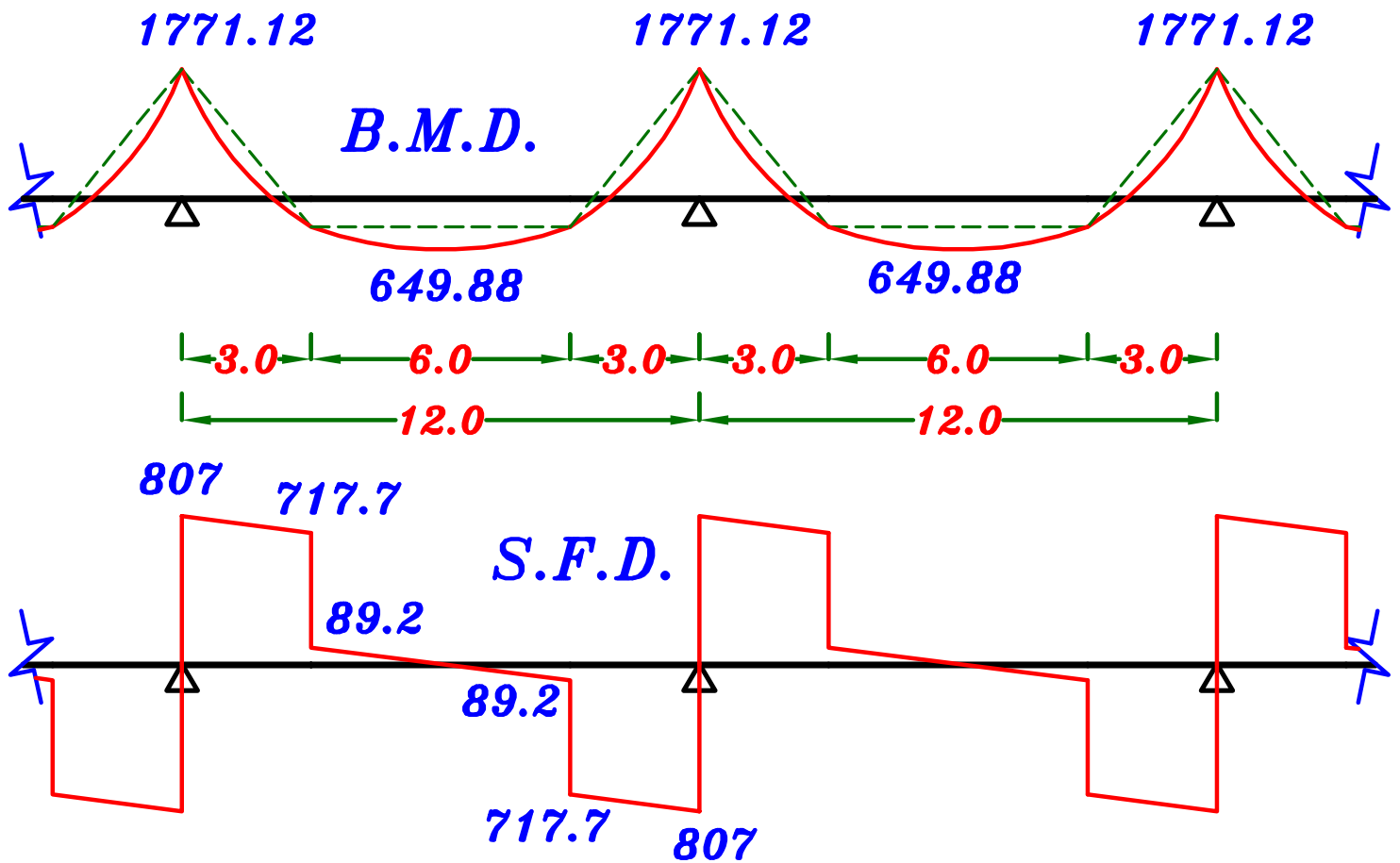
Loads on Girder.



$$\begin{aligned} \text{Fixed End Moment} &= \frac{wL^2}{12} + \frac{Pa(b)^2}{L^2} + \frac{Pb(a)^2}{L^2} \\ &= \frac{29.75 * 12^2}{12} + \frac{628.5 * 3 * 9^2}{12^2} + \frac{628.5 * 9 * 3^2}{12^2} = 1771.12 \text{ kN.m} \end{aligned}$$



$$\begin{aligned} B.M. (\text{at mid. span}) &= 807(6.0) - 1771.12 - 29.75 * 6.0(3.0) - 628.5(3.0) \\ &= 649.88 \text{ kN.m} \end{aligned}$$



Torsion on Girder.

$$M_t = 628.5 * 0.60 = 377.1 \text{ kN.m}$$

